0-π and 0-κ Josephson junctions: from fractional vortices to tunable current-phase relation

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- Prof.A. Buzdin

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(nonlinear theory)
- Prof. H. Susanto

Univ. Tel-Aviv:
(nonlinear theory)
- Prof. R. Mints

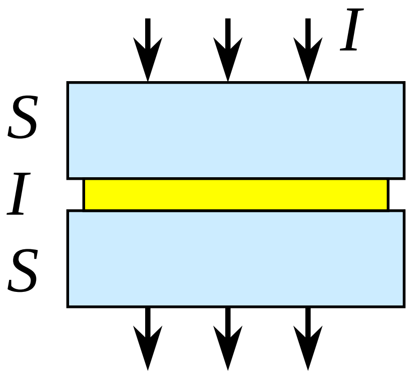
Univ. Tokio:
(quantum theory)
- Prof. T. Kato

Plan

- 0 JJ + π JJ = 0-π JJ.
- Technologies
  - SIFS 0-π JJs
  - s-wave/s-wave 0-π JJs
  - creating artificial phase discontinuities, 0-κ junction.
- Single fractional vortex
  - ground states
  - depinning by bias current, thermal escape, MQT
  - eigenmodes
- Fractional vortex molecules
  - ground states
  - rearrangement by bias current
  - eigenmodes splitting
- φ Josephson junctions and tunable CPR
- Conclusions and outlook

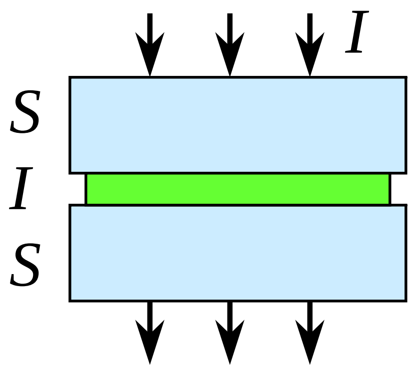
0 & π Josephson junctions

Conventional JJ. (0-junction)

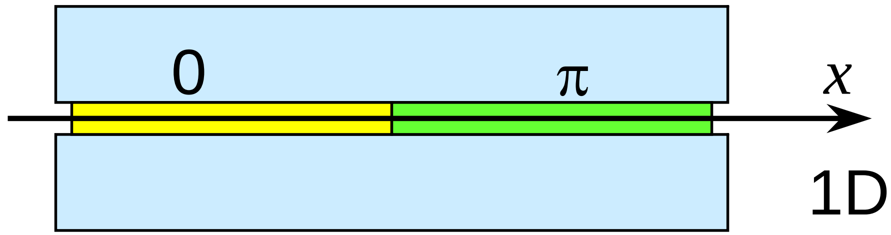


$$I = I_c \sin(\phi)$$

Unconventional JJ (π-junction)



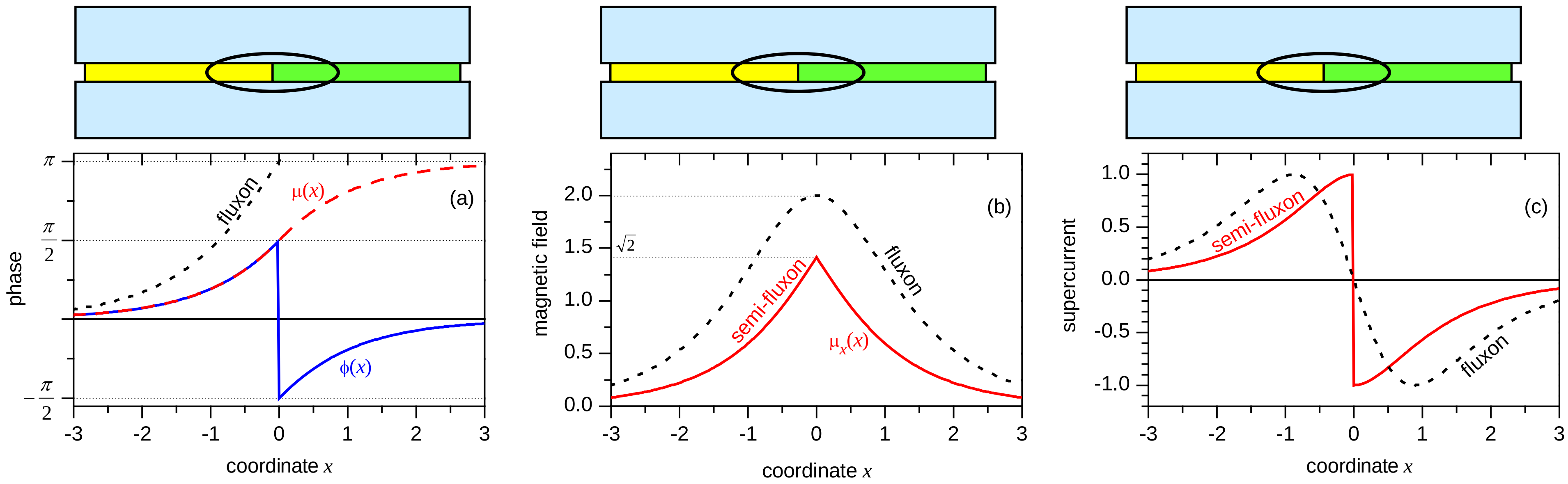
$$I = -I_c \sin(\phi) = I_c \sin(\phi + \pi)$$



1D model

Ground state?

Semifluxon=vortex carrying $\pm\Phi_0/2$

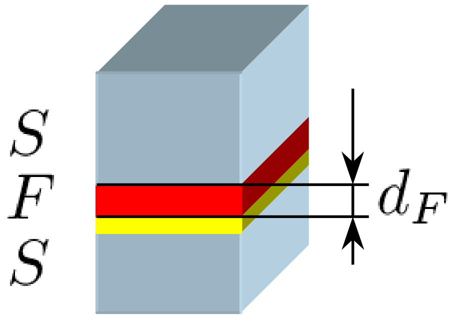


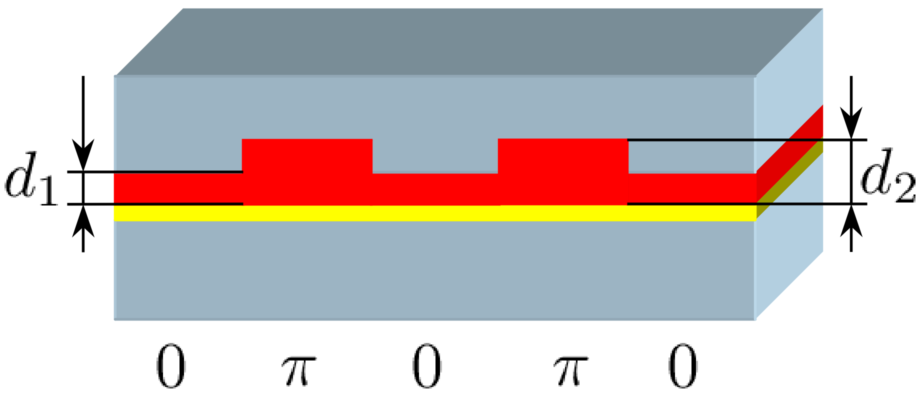
Semifluxon properties

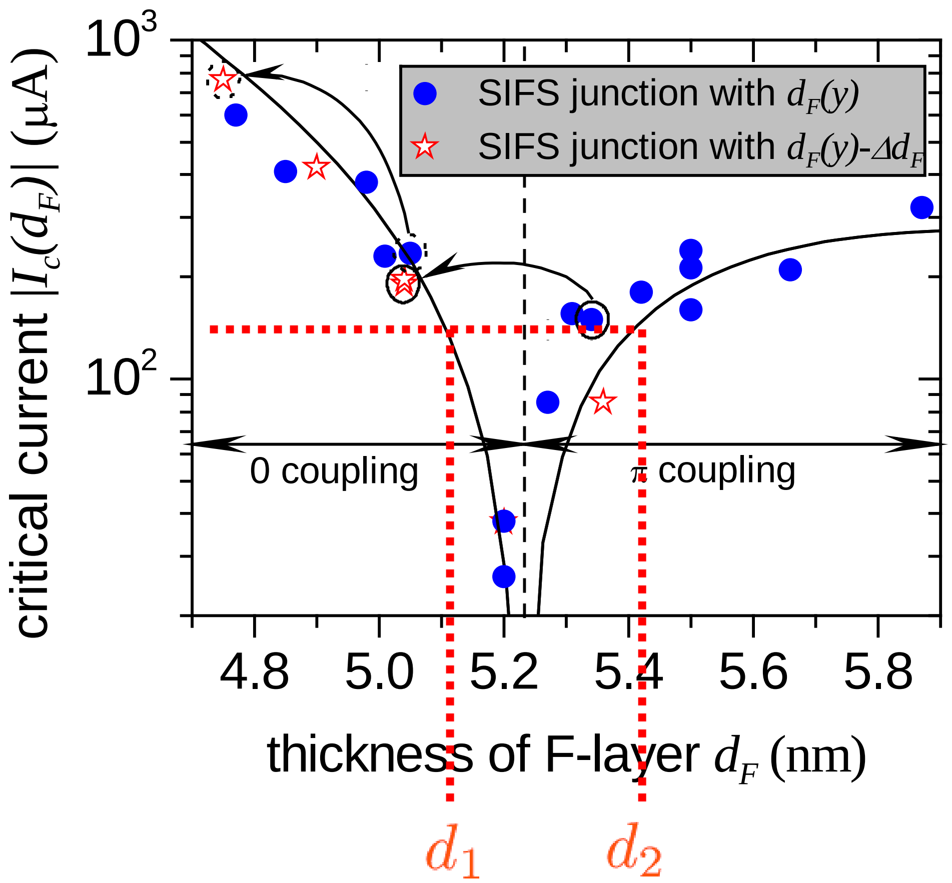
- pinned at 0-π boundary
- has two degenerate ground states
  - state ↑ ($+\Phi_0/2$, supercurrent clockwise)
  - state ↓ ($-\Phi_0/2$, supercurrent counterclockwise)

📖 L. Bulaevskii et al. Solid State Comm. **25**, 1053 (1978);
📖 Xu et al., Phys. Rev. B **51**, 11958 (1995);
📖 Goldobin et al., Phys. Rev. B **66**, 100508 (2002).

SIFS 0-π junctions



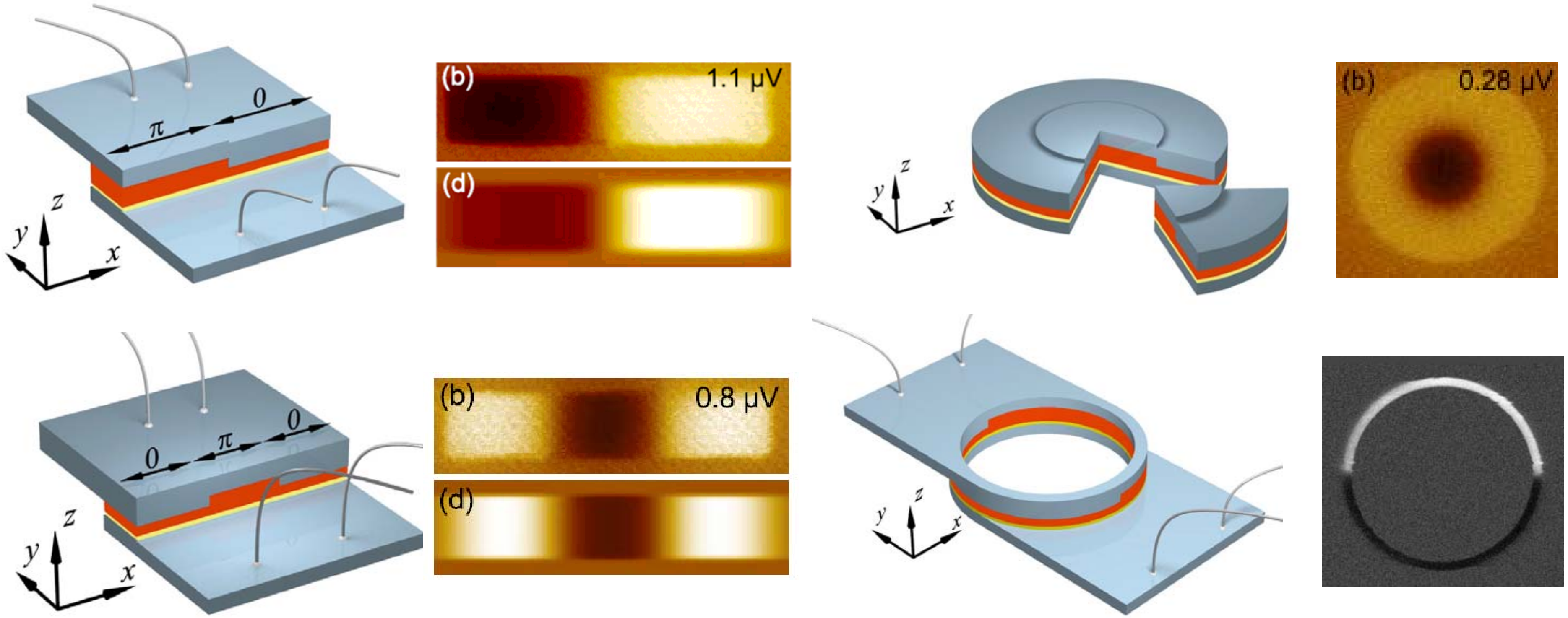




M. Weides et al., PRL **97**, 247001 (2006)

T. Kontos et al. PRL **89**, 137007 (2002)

State of the art SIFS technology

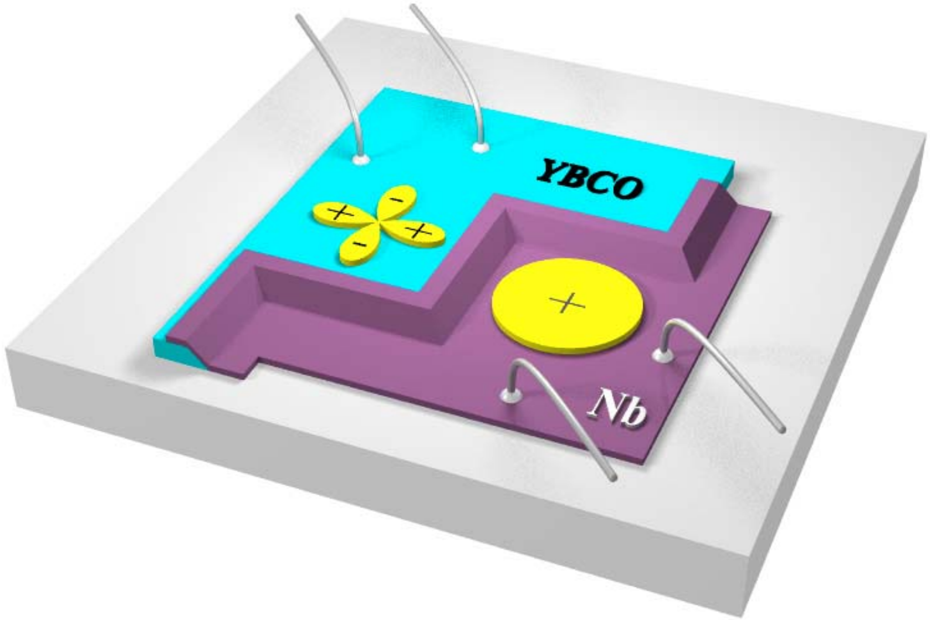


- current densities j_c in π state up to 40 A/cm^2
- arbitrary topology:
  - torus-like half-flux lines,
- Attention: j_c in 0 and π parts are not equal
- Future: higher j_c using e.g. clean ferromagnet

Ch. Gürlich et al., PRB **81**, 094502 (2010)

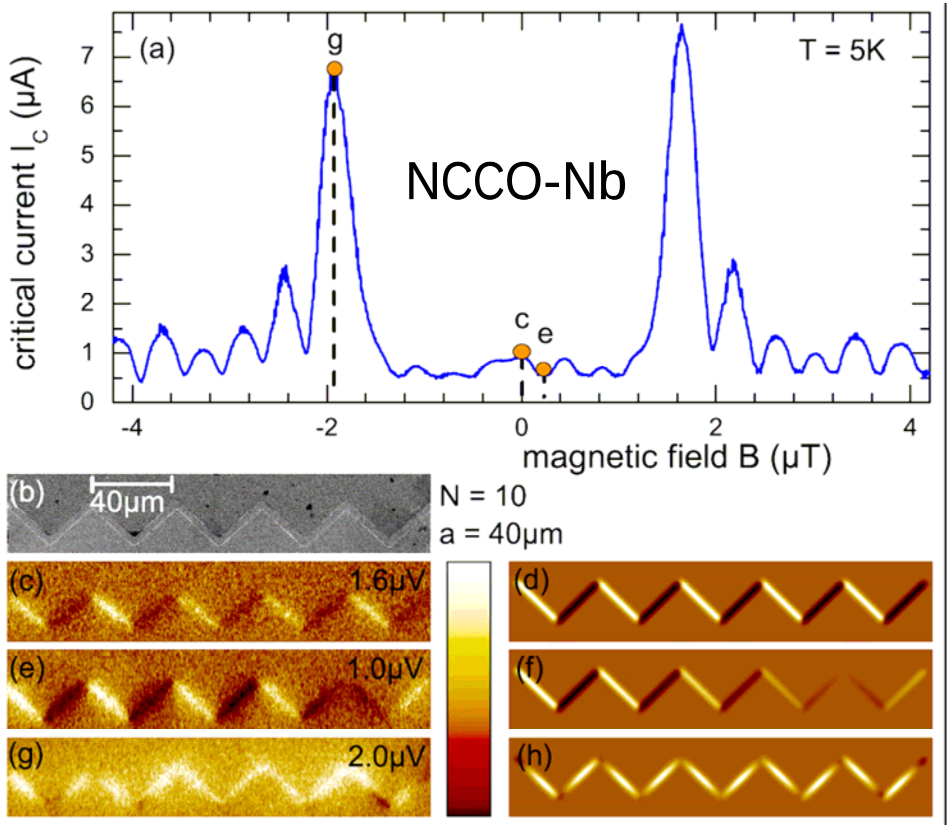
d-wave/s-wave 0-π JJs

D-wave/s-wave ramp zigzag JJ:



📖 YBCO-Nb:
H.-J. Smilde et al. PRL **88**, 57004 (2002)
📖 NCCO-Nb:
Ariando et al., PRL 94, 167001 (2005)

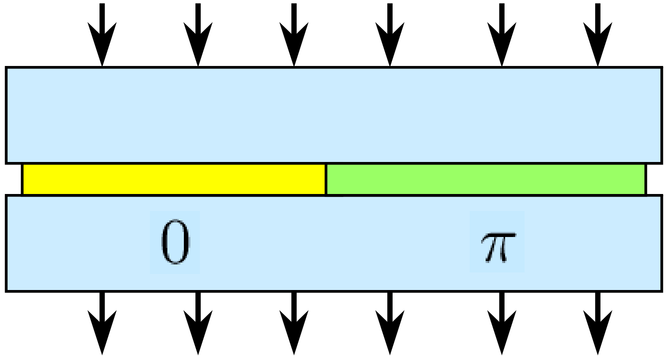
Short facets

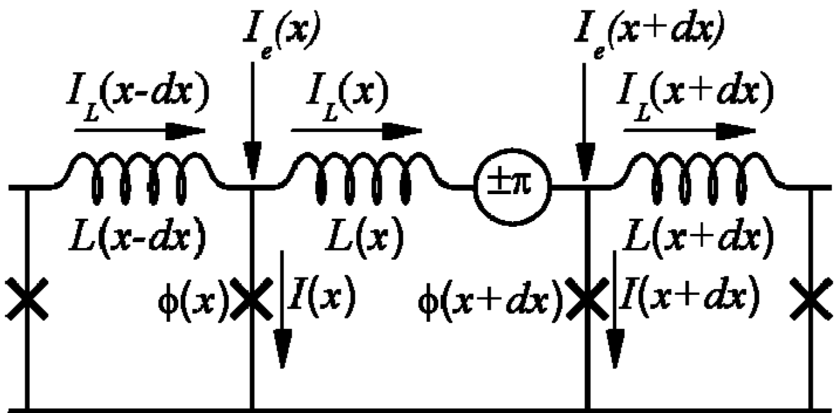


LTSEM images of supercurrent

📖 Ch. Guerlich et al., PRL **103**, 067011 (2009)

Deriving sine-Gordon equation





$$I = j(x) w\, dx, \quad I_e = j_e(x) w\, dx, \quad L = \frac{\mu_0 d'}{w} dx,$$

$$\Phi_e = \mu_0 (\mathbf{H}\cdot\mathbf{n}) \Lambda dx = \mu_0 H(x) \Lambda\, dx$$

$$\phi(x+dx) - \phi(x) = \frac{2\pi}{\Phi_0}[\Phi_e - I_L(x) L(x)] + \Pi(x),$$

$$I_L(x) + I_e(x) = I_L(x+dx) + I(x),$$

$$\Pi(x) = \theta(x+dx) - \theta(x)$$, where

$$\theta(x) = \pi \sum_{k=1}^{N_c} \sigma_k \mathcal{H}(x - x_k),$$

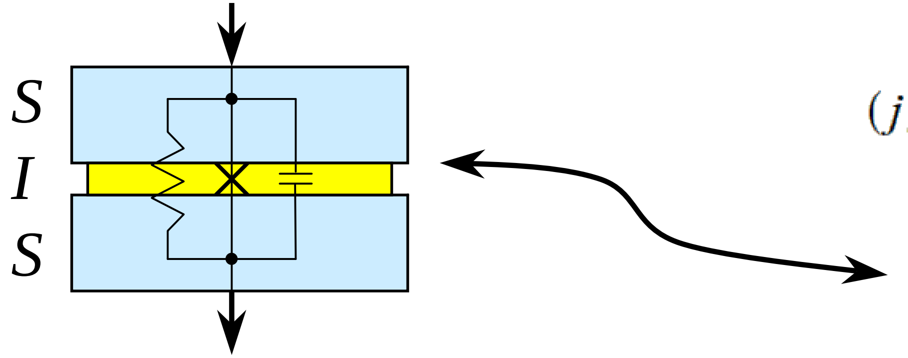
$$\phi_x = \frac{2\pi}{\Phi_0}\left[H\Lambda - \frac{I_L}{\mu_0 d'} \right] + \theta_x(x),$$

$$\frac{dI_L}{dx} = (j_e - j) w.$$

Exclude I_L...

E. Goldobin et al. PRB **66**, 100508 (2002)

Deriving sine-Gordon equation



$$(j_e - j) = \frac{1}{\mu_0 d'}\left\{\mu_0 \Lambda H_x(x) - \frac{\Phi_0}{2\pi}[\phi_{xx} - \theta_{xx}(x)]\right\}$$

$$j(x) = j_c \sin(\phi) + \frac{\Phi_0}{2\pi\rho}\phi_t + C'\frac{\Phi_0}{2\pi}\phi_{tt}$$

$$\lambda_J^2 \phi_{xx} - \omega_p^{-2}\phi_{tt} - \sin(\phi) = \omega_c^{-1}\phi_t - \gamma(x) + QH_x(x) + \lambda_J^2 \theta_{xx}(x),$$

$\lambda_J = \sqrt{\Phi_0/(2\pi\mu_0 j_c d')}$ → Josephson penetration depth ~0.3–100μm

$\omega_p = \sqrt{2\pi j_c/(\Phi_0 C')}$ → Josephson plasma frequency ~10–10^3GHz

$\omega_c = 2\pi j_c \rho/\Phi_0$ → Josephson critical frequency ~1–10^5 GHz

$\gamma(x) = j_e(x)/j_c$ → normalized bias current density

$Q = 2\pi\mu_0\Lambda\lambda_J^2/\Phi_0$

New normalized units:coordinate x to λ_J, time t to ω_p^{-1}

📖 E. Goldobin et al. PRB **66**, 100508 (2002)

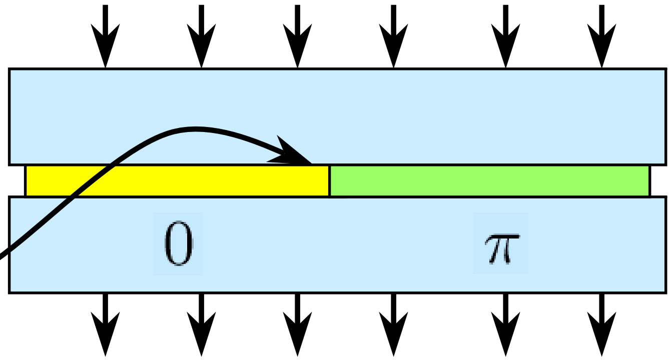
sine-Gordon equation for 0-π LJJ

$$\phi_{xx} - \phi_{tt} - \sin(\phi) = \alpha\phi_t - \gamma(x) + h_x(x) + \theta_{xx}(x)$$

$\alpha = \omega_p / \omega_c \equiv 1/\sqrt{\beta_c}$ — dimensionless damping

$h(x) = 2H(x)/H_{c1}$ — dimensionless field

$H_{c1} = \Phi_0 / (\pi \mu_0 \Lambda \lambda_J)$ — the first critical field



Phase discontinuity points!

$$\phi(x,t) = \mu(x,t) + \theta(x).$$

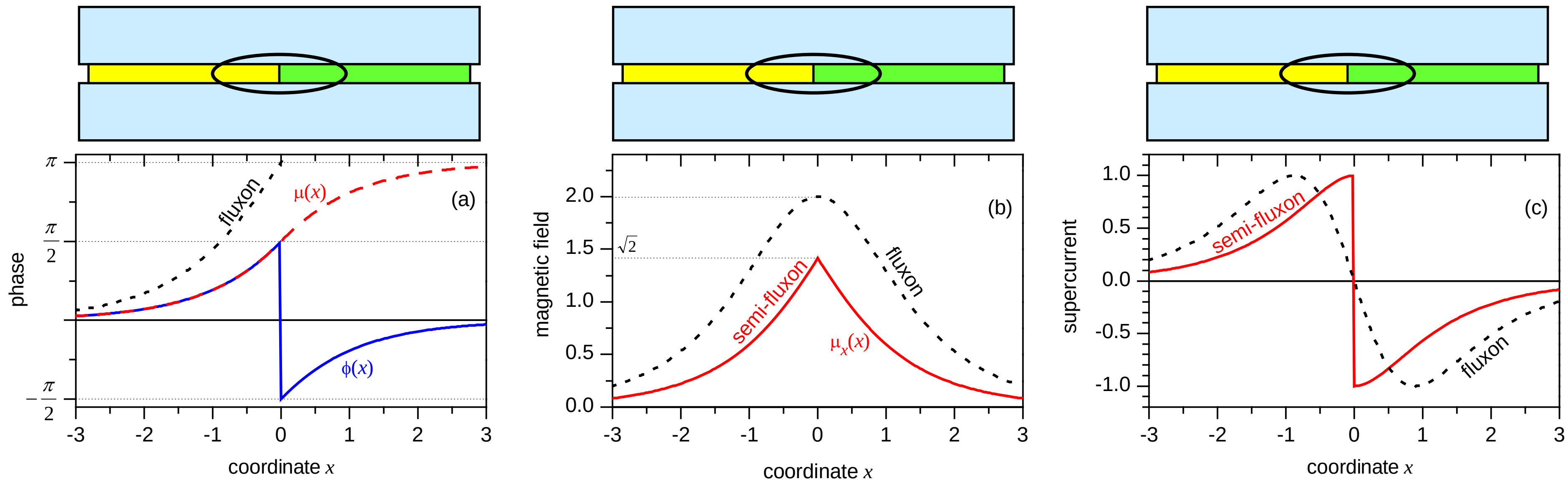
$\mu(x,t)$ — magnetic component of the phase

$$\mu_{xx} - \mu_{tt} - \sin(\mu) \underbrace{\cos(\theta)}_{\pm 1} = \alpha\mu_t - \gamma(x) + h_x(x).$$

📖 Goldobin et al., PRB **66**, 100508 (2002)

Semifluxon=vortex carrying $\Phi_0/2$

$$\phi_{xx} - \sin(\phi) = \theta_{xx}(x)$$



$$\phi(x) = -4\,\mathrm{sign}(x)\arctan\left(\mathcal{G}e^{-|x|}\right),\qquad \mu_x(x) = \frac{2}{\cosh(|x| - \ln\mathcal{G})}$$

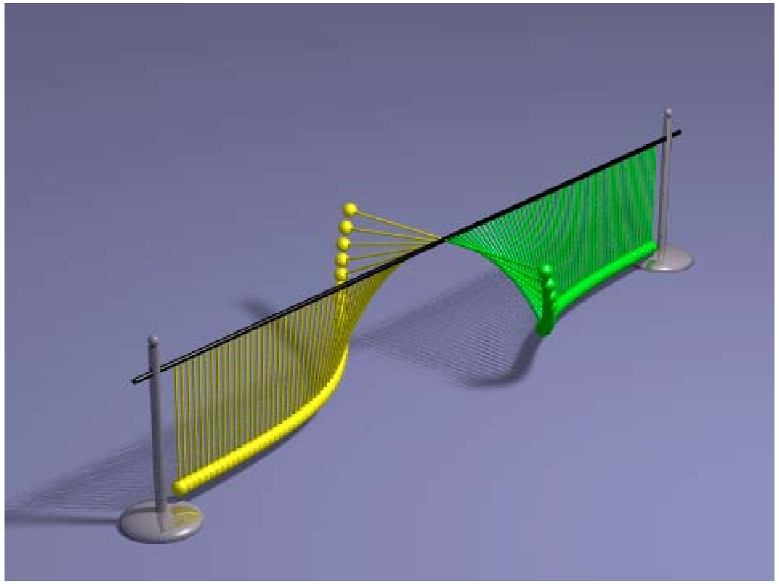
$$\mathcal{G} = \tan(\pi/8) = \sqrt{2} - 1 \approx 0.4$$

Pinned, two degenerate states ↑ and ↓

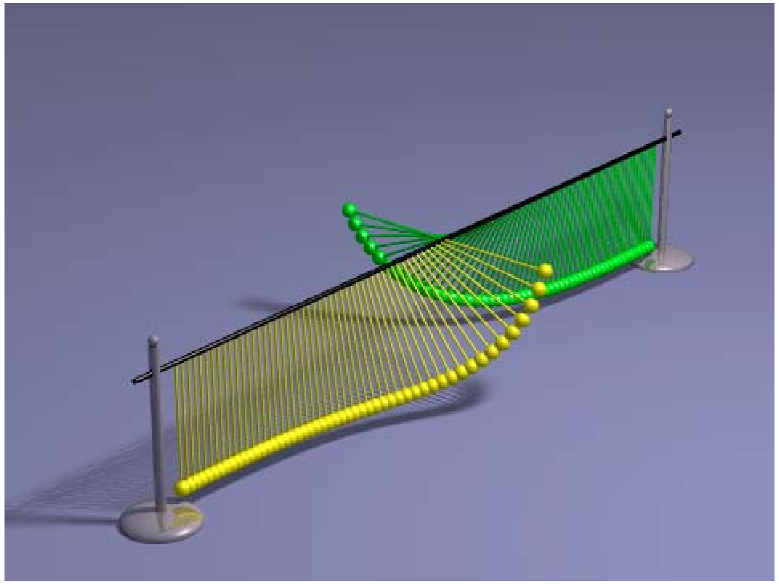
📖 Xu et al., PRB **51**, 11958 (1995)

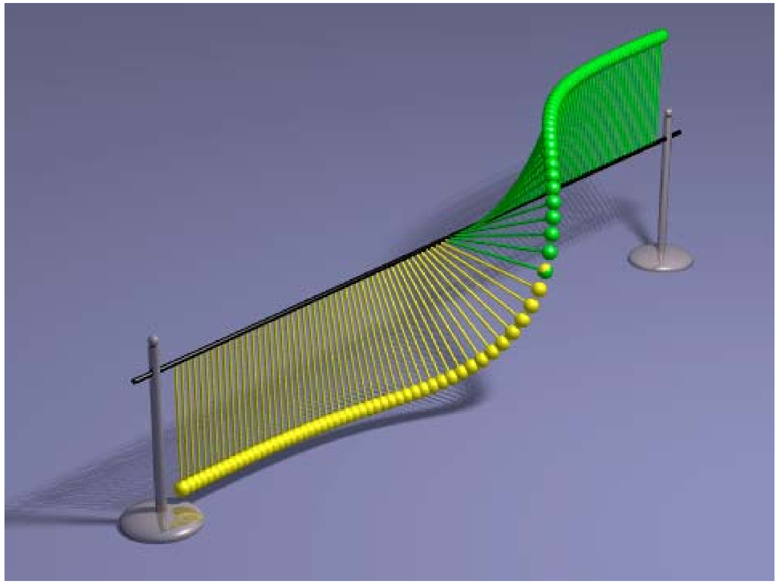
📖 Goldobin et al., PRB **66**, 100508 (2002)

Mechanical analog:pendula chain

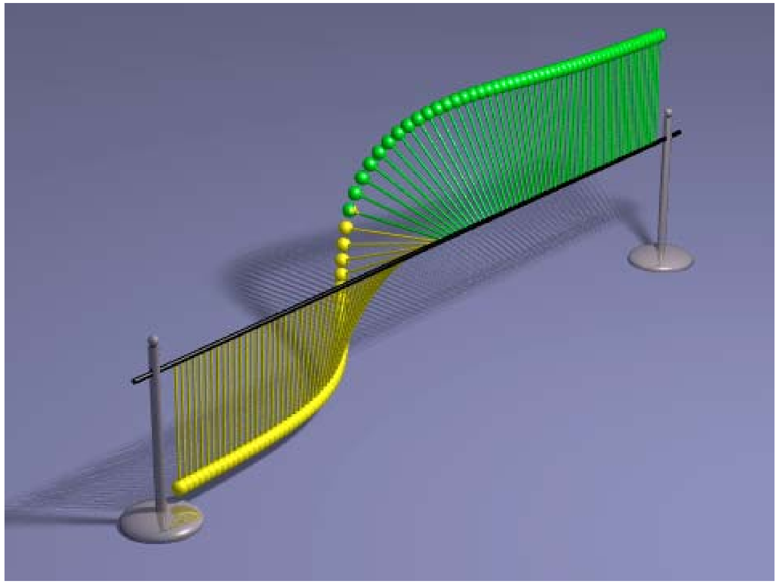


$\phi(x)$

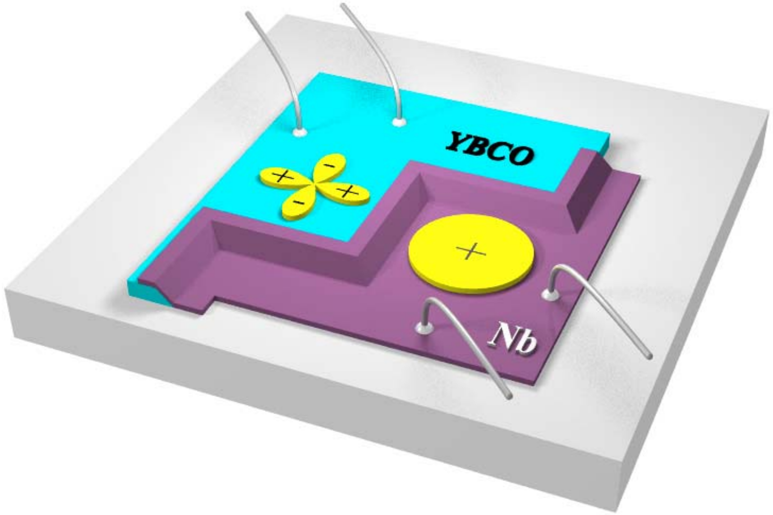




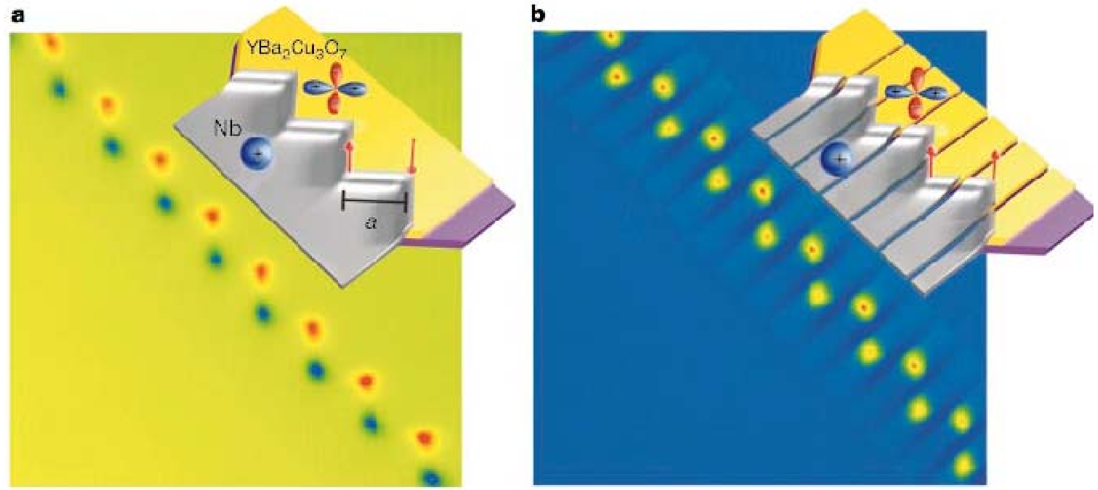
$\mu(x)$



Semifluxons observation



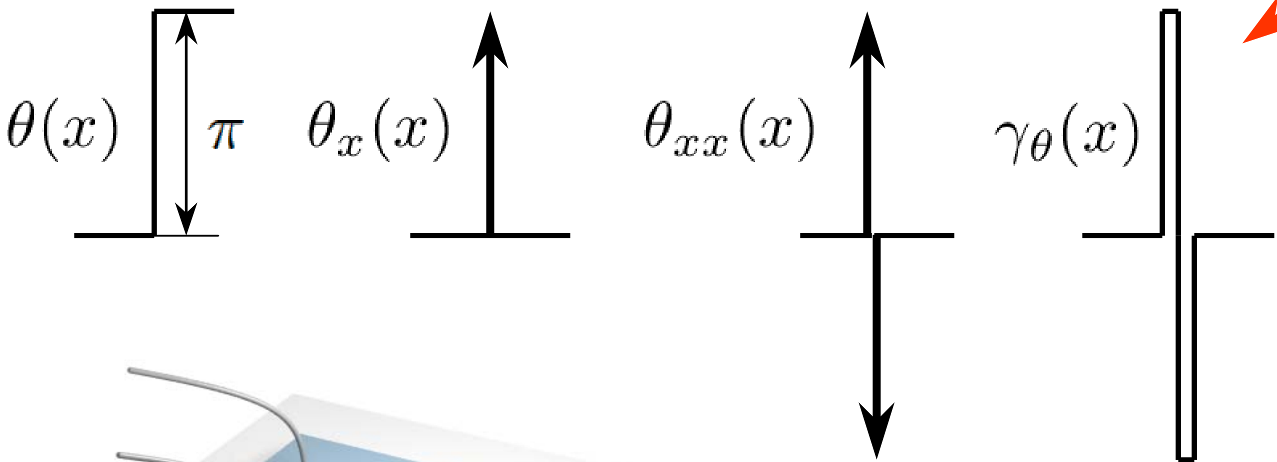
SQUID microscopy on YBCO-Nb ramp zigzag LJJs



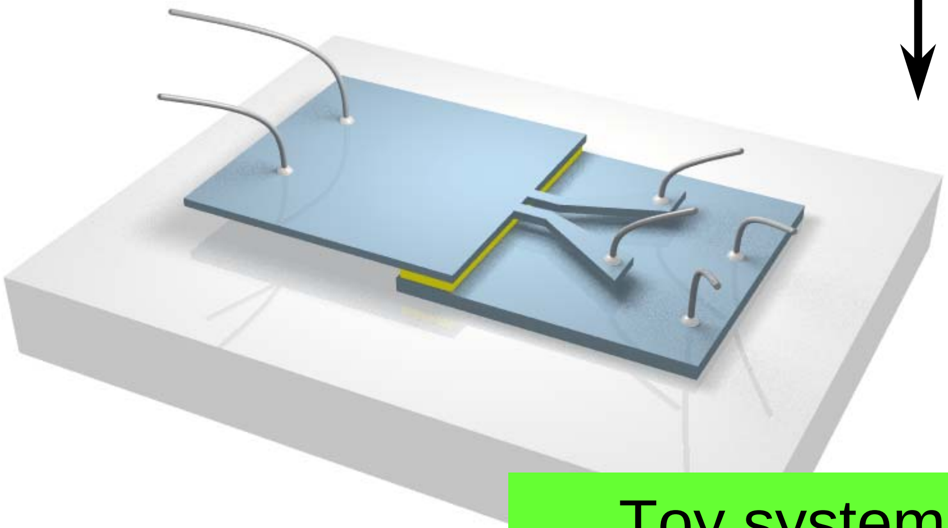
H. Hilgenkamp et al. Nature **422**, 50 (2003).

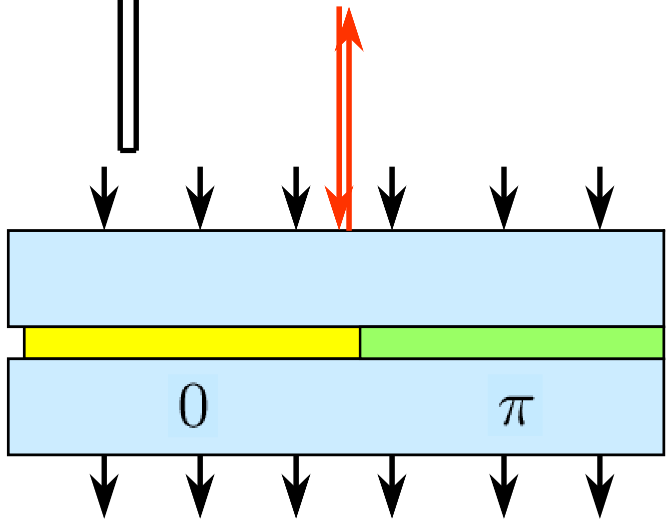
Artificial 0-π or 0-κ junction

$$\phi_{xx} - \phi_{tt} - \sin(\phi) = \alpha\phi_t - \gamma(x) + \theta_x(x)$$



α - damping

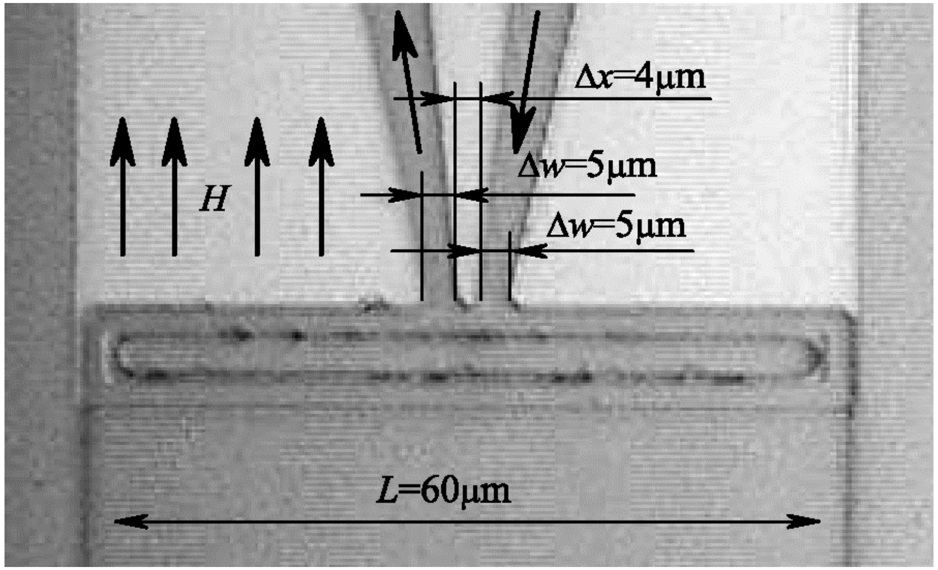




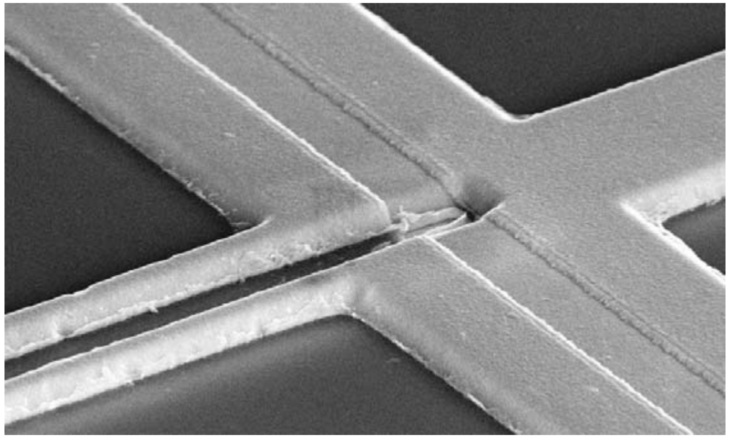
Toy system to study arbitrary fractional vortices!

📖 Goldobin et al., PRL **92,** 057005 (2004); see also the talk of M. Merker (Mo 9:45)

Nb LJJ with two injectors



$\lambda_J \approx 30\ \mu m$ ($j_c \approx 100$ A/cm^2)

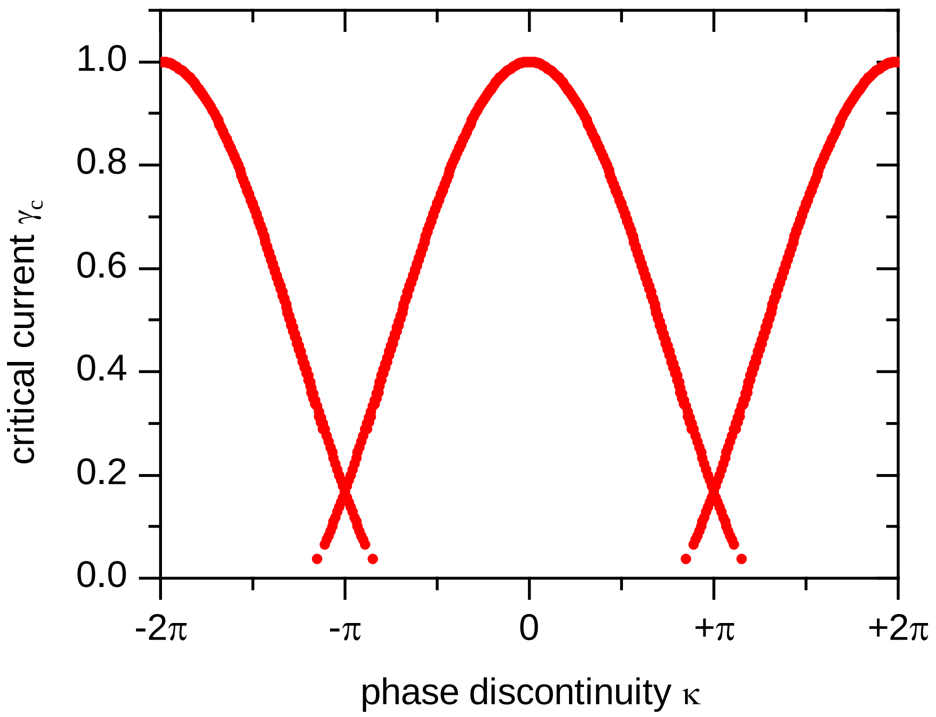


$\lambda_J \approx 8\ \mu m$ ($j_c \approx 4$ kA/cm^2)
$\Delta w_{\rm inj}$=500nm, $\Delta x_{\rm inj}$=500nm, w=700nm

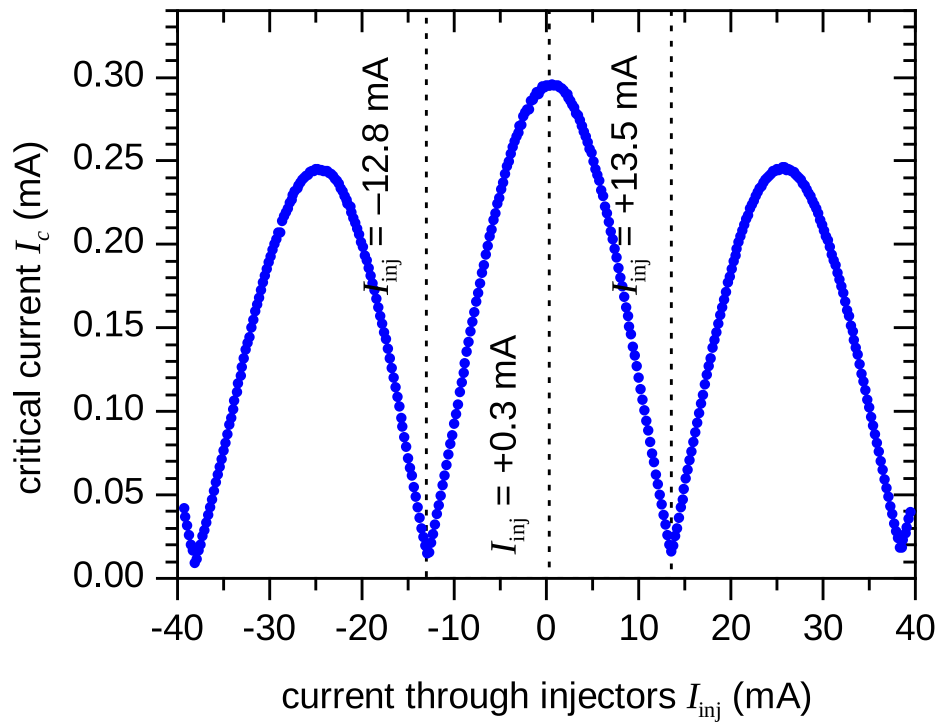
Meckbach et al. arXiv:1210.1012

Calibration of injectors

Numerical

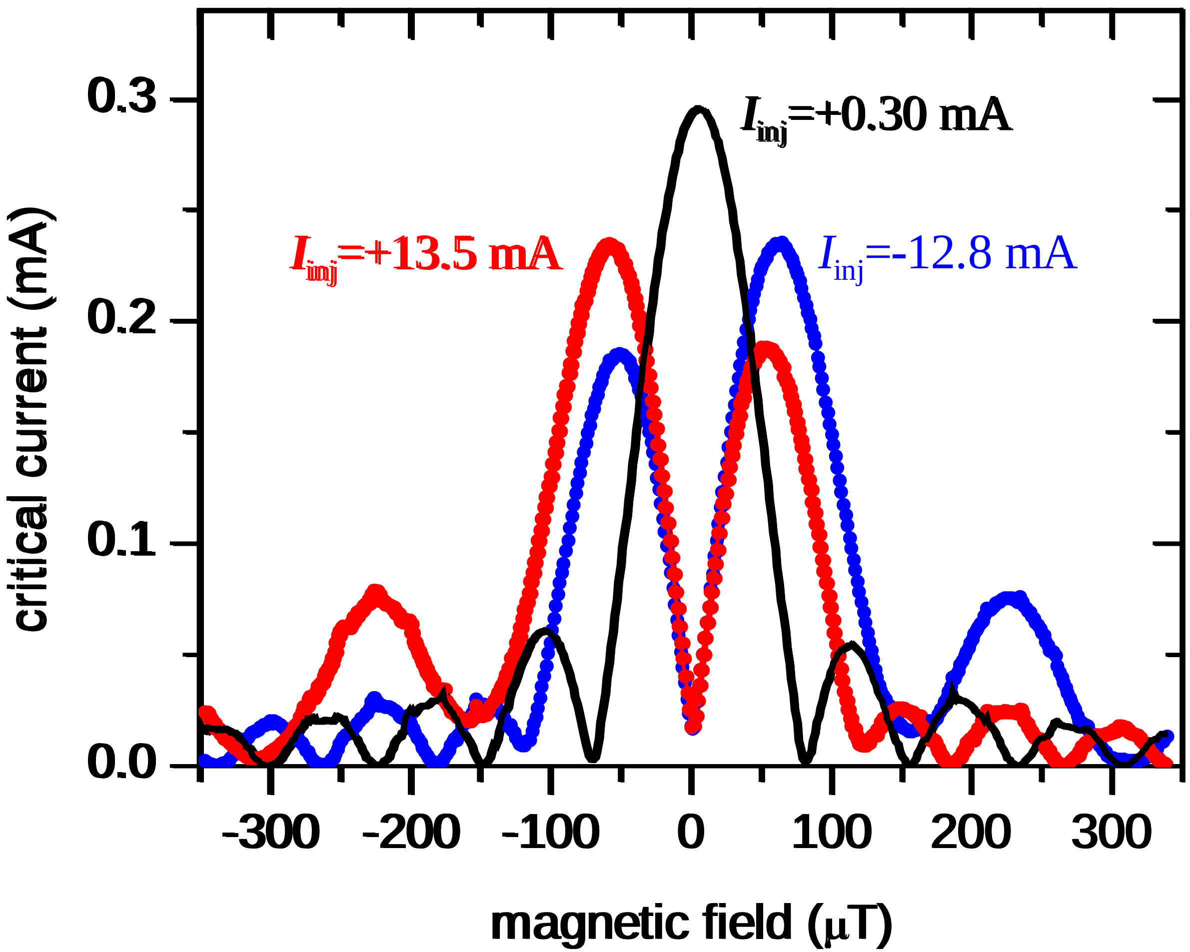


Experimental



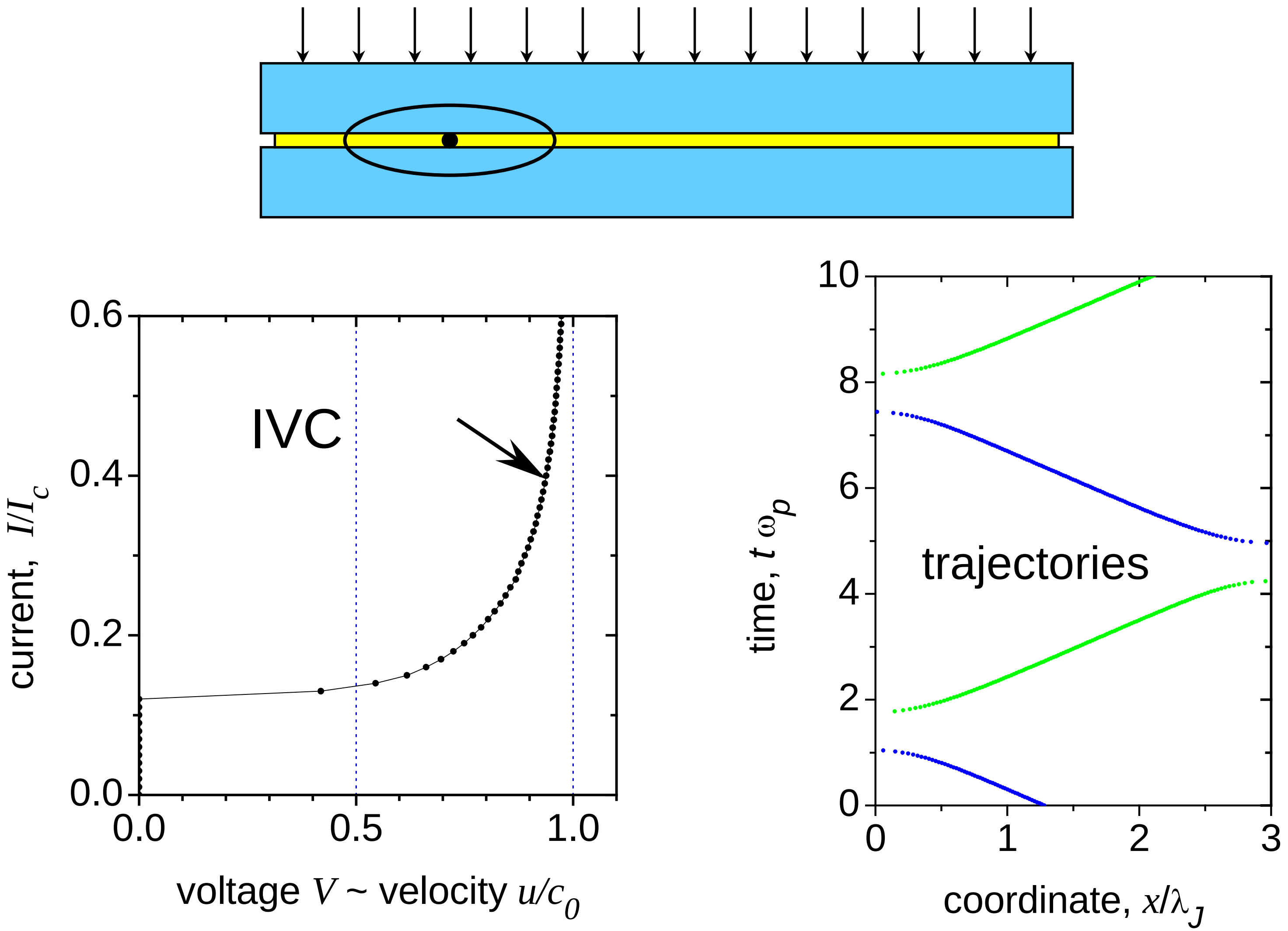
Goldobin et al., PRL **92**, 057005 (2004)

$I_c(H)$ in 0-0 and in 0-π states



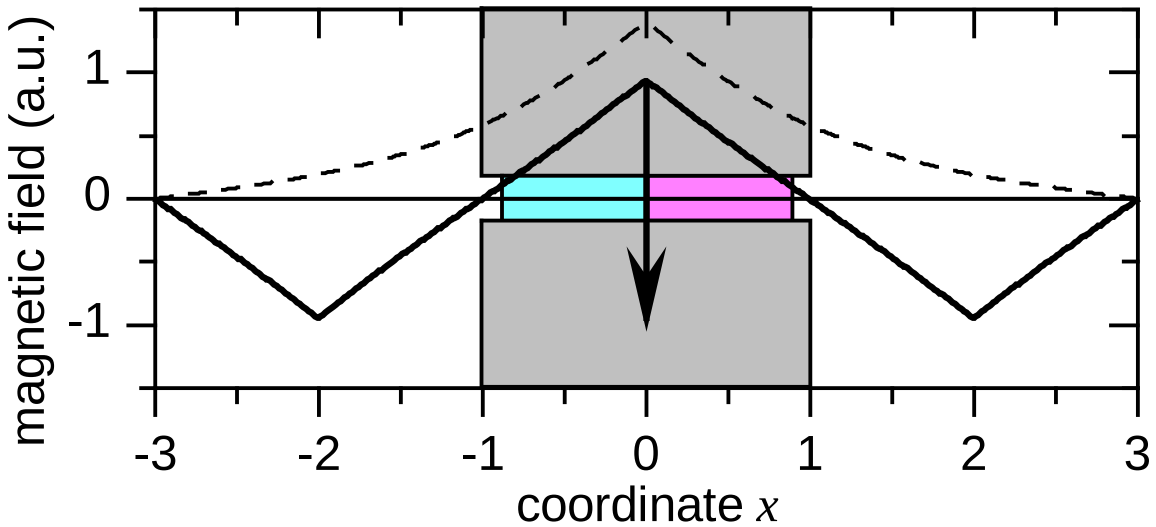
Goldobin et al., PRL **92**, 057005 (2004)

Classical Zero Field Step (ZFS)



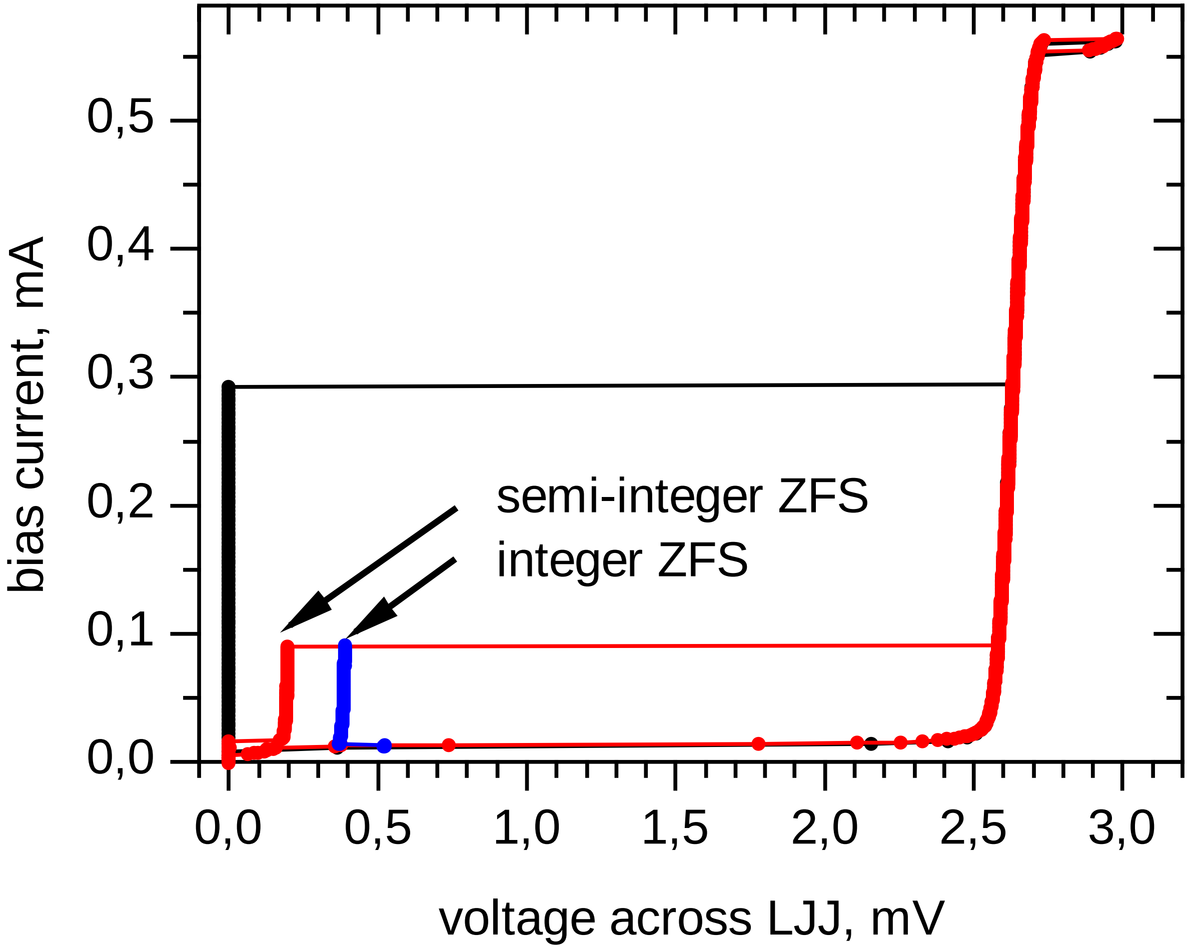
Semifluxon -- half-integer ZFS

- Finite length --> image technique:
  - 1 real semifluxon + 2 anti-semifluxons (images)
- Bias current → Force → SF hopping.



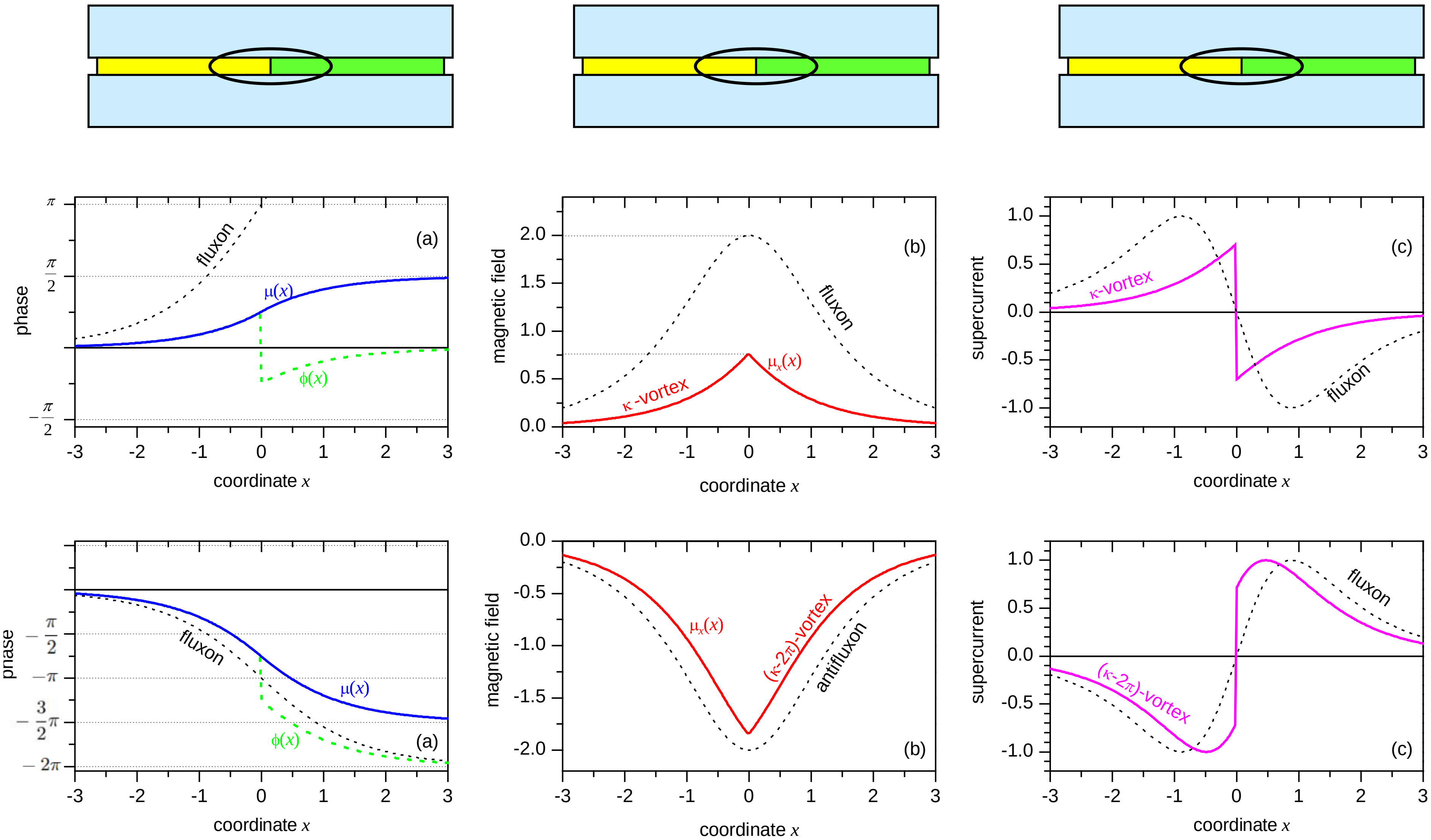
Goldobin et al., Phys. Rev. B **67**, 224515 (2003).

Half integer ZFS (full IVC)



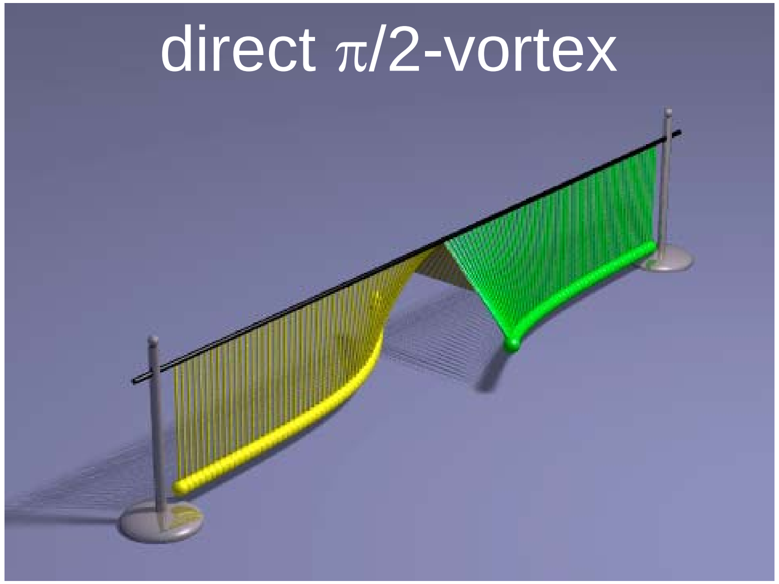
Goldobin et al., PRL **92**, 057005 (2004)

κ-vortex: broken symmetry

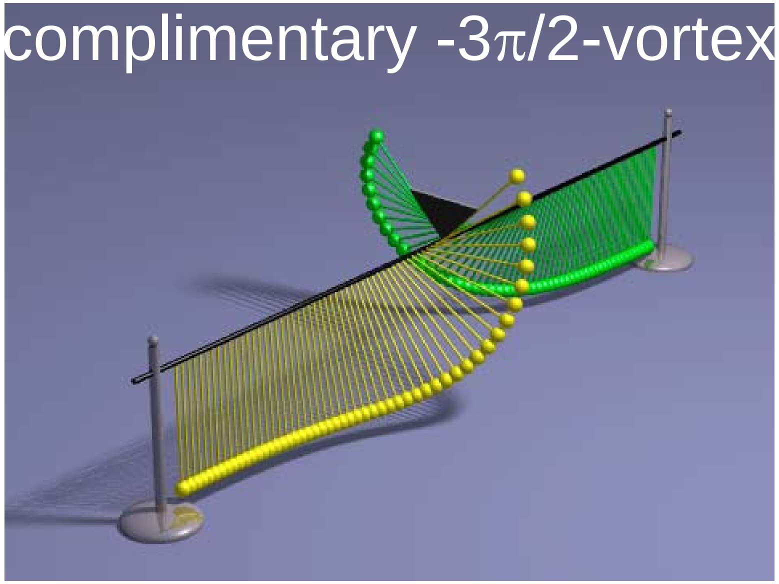


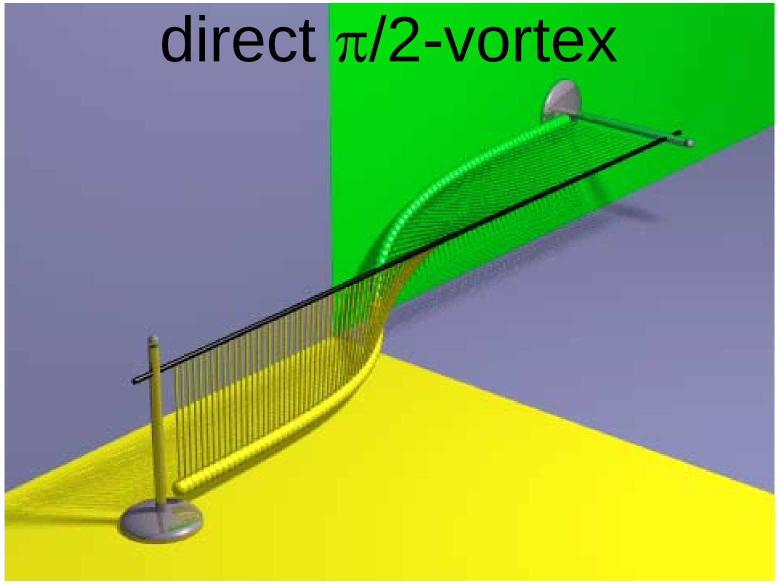
Goldobin et al., PRB **70**, 174519 (2004)

Fractional vortices at $\kappa=\pi/2$

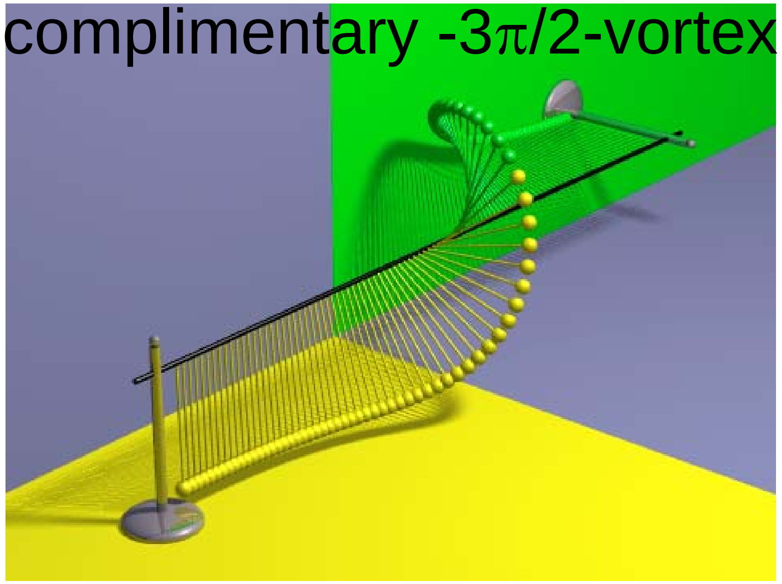


$\phi(x)$



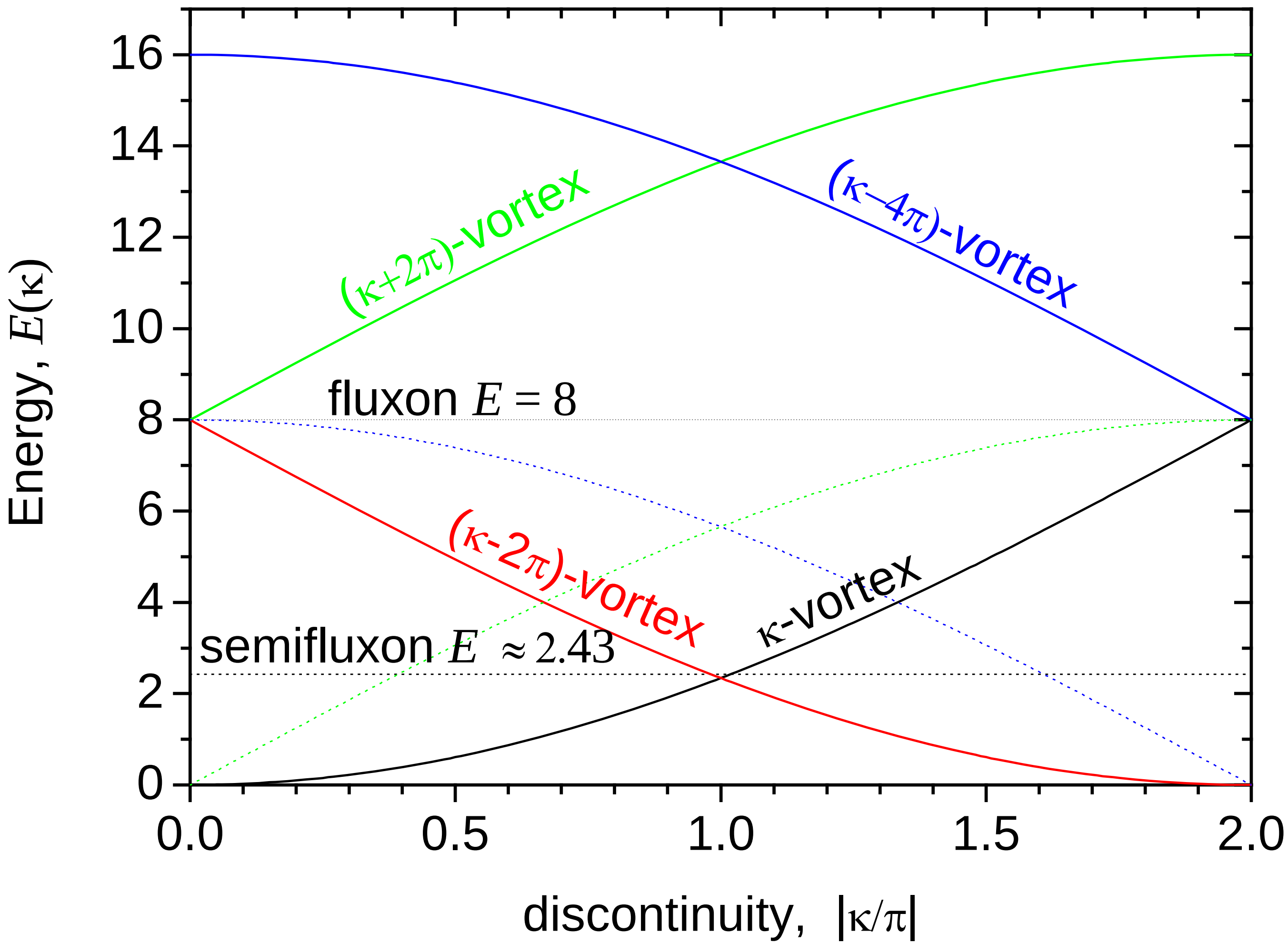


$\mu(x)$



Energy of a single vortex

$$E(\kappa) = 16 \sin^2\left(\frac{\kappa}{8}\right)$$

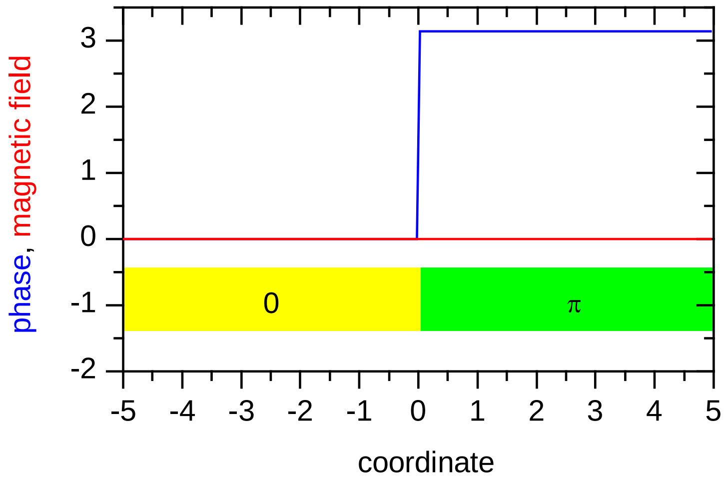


Goldobin et al., Phys. Rev. B **70**, 174519 (2004)

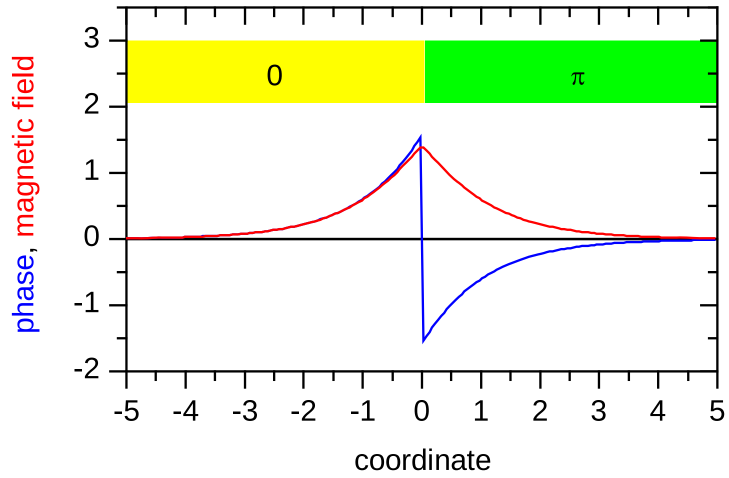
0-π boundary: semifluxon vs. μ=0

$$\phi_{xx} - \sin(\phi) = \theta_{xx}(x)$$

flat phase state



semifluxon



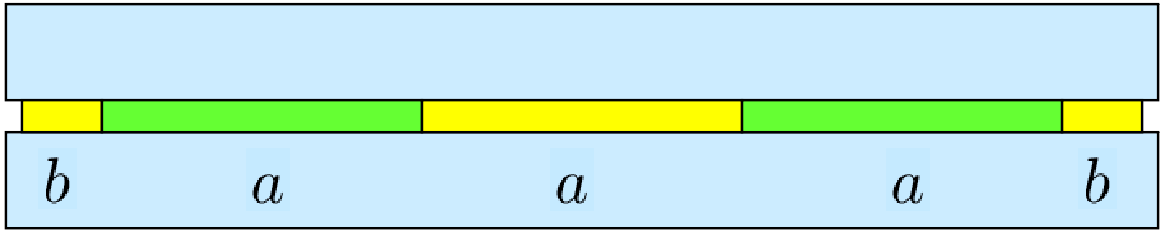
The energy of flat phase is 2 per unit of length, i.e. diverges at large *L*.

$$U = 16\frac{\mathcal{G}^2}{1+\mathcal{G}^2} = 8 - 4\sqrt{2} \approx 2.343$$

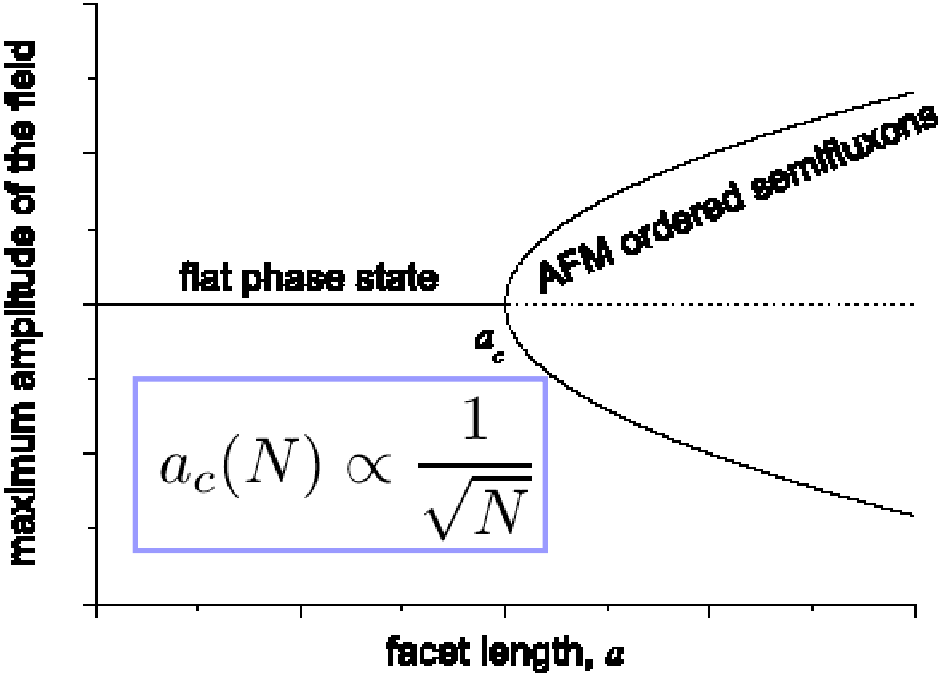
- One 0-π-boundary:
  - always semifluxon
  - flux-less flat phase solution (0-π) is unstable (has infinite energy)!

E. Goldobin et al. Phys. Rev. B **67**, 224515 (2003)

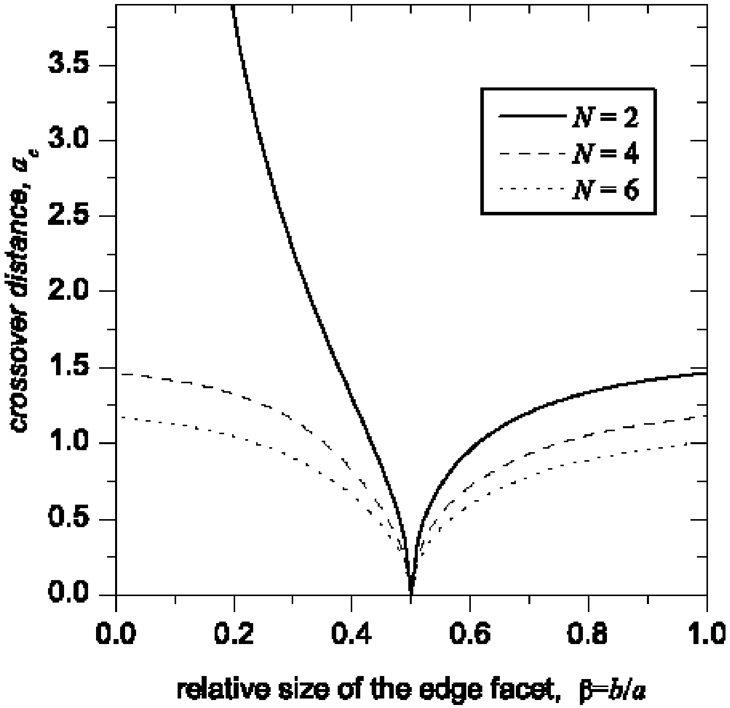
Behavior of $a_c^{(N)}(b)$



For odd N $a_c^{(N)} = 0$.



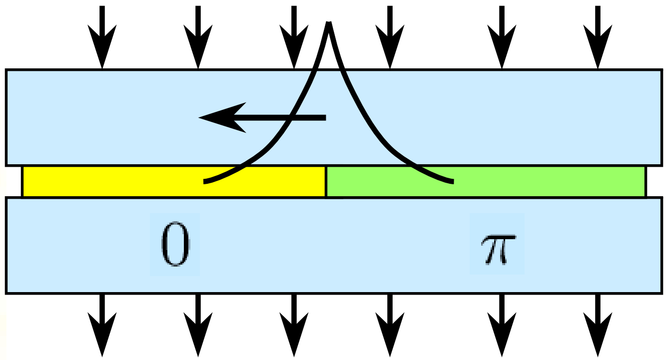
$$a_c(N) \propto \frac{1}{\sqrt{N}}$$

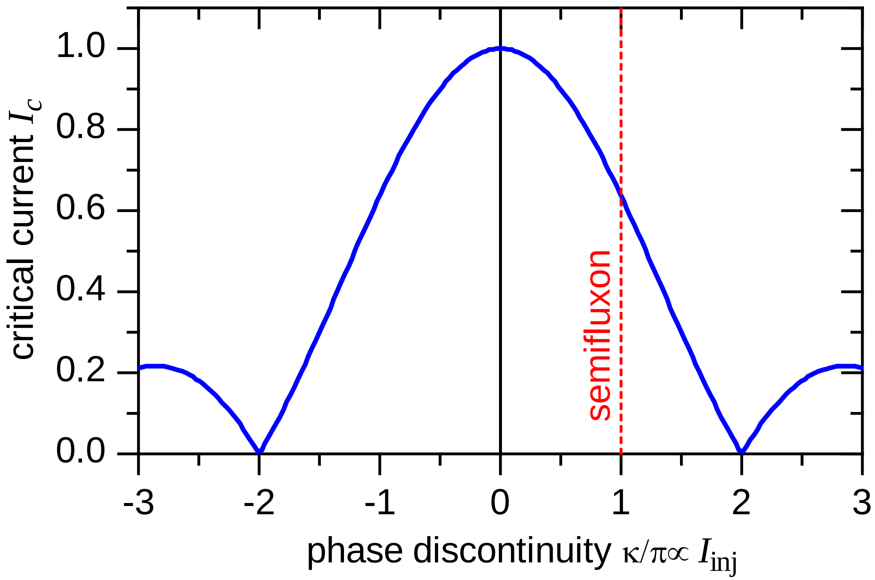


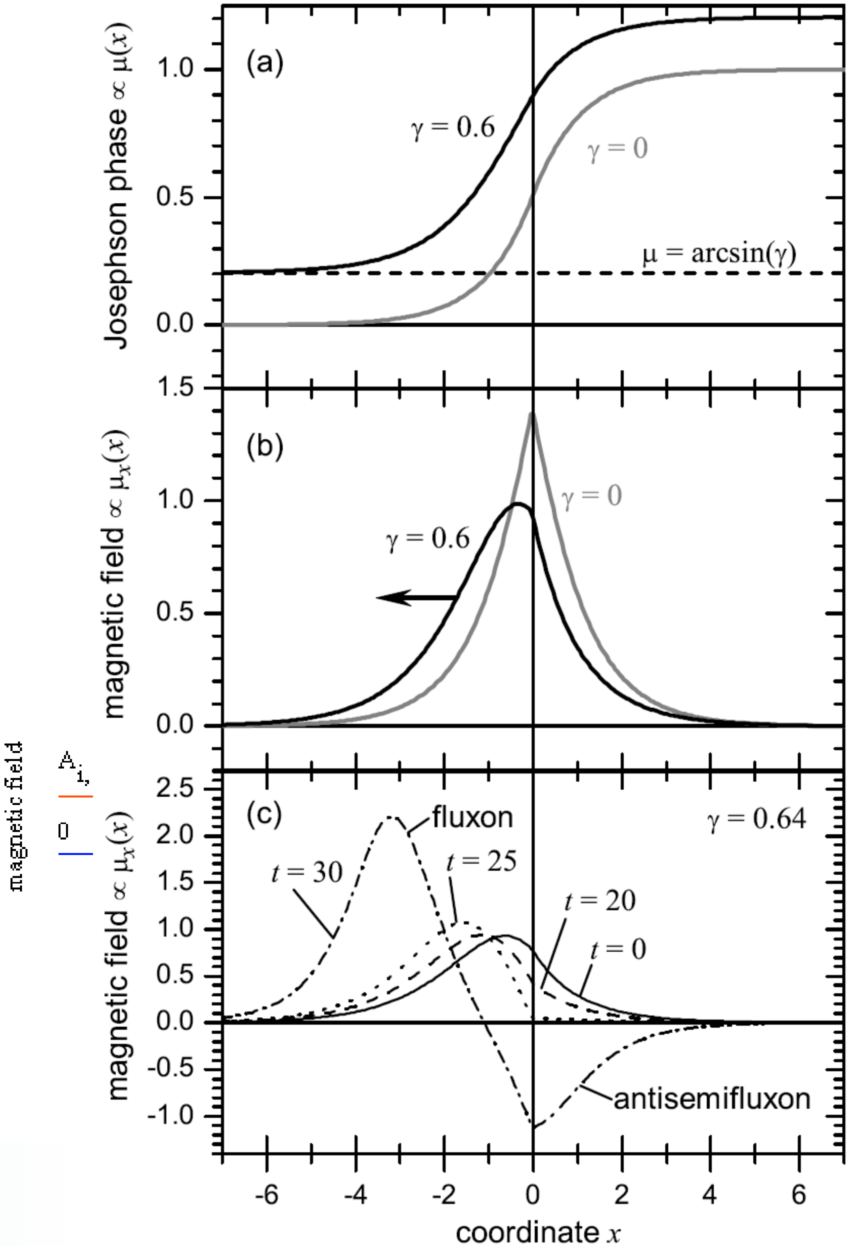
$$a_c^{(2)} \approx 1.55 \pm 0.05 \qquad a_c^{(4)} = 1.35 \pm 0.05 \qquad a_c^{(6)} = 1.15 \pm 0.05$$

📖 A. Zenchuk, E. Goldobin, PRB **69**, 024515 (2004).

Switching on the current...

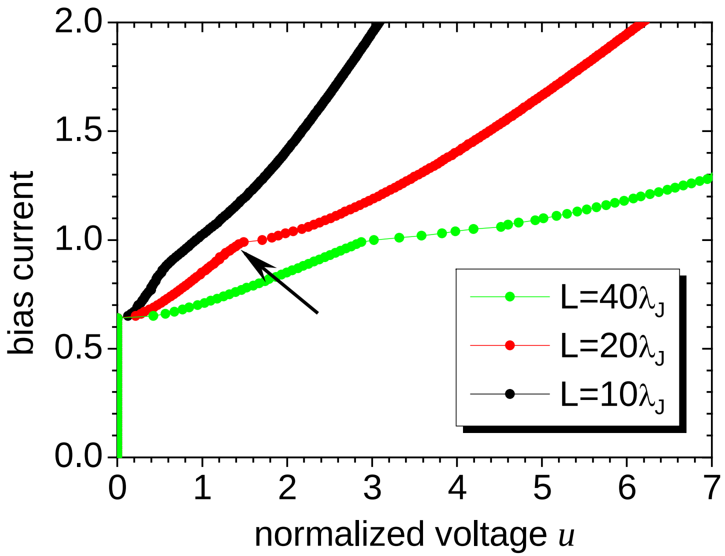


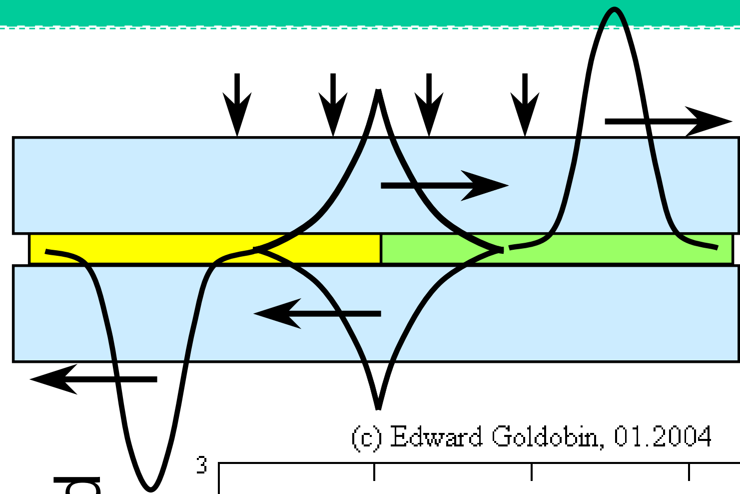


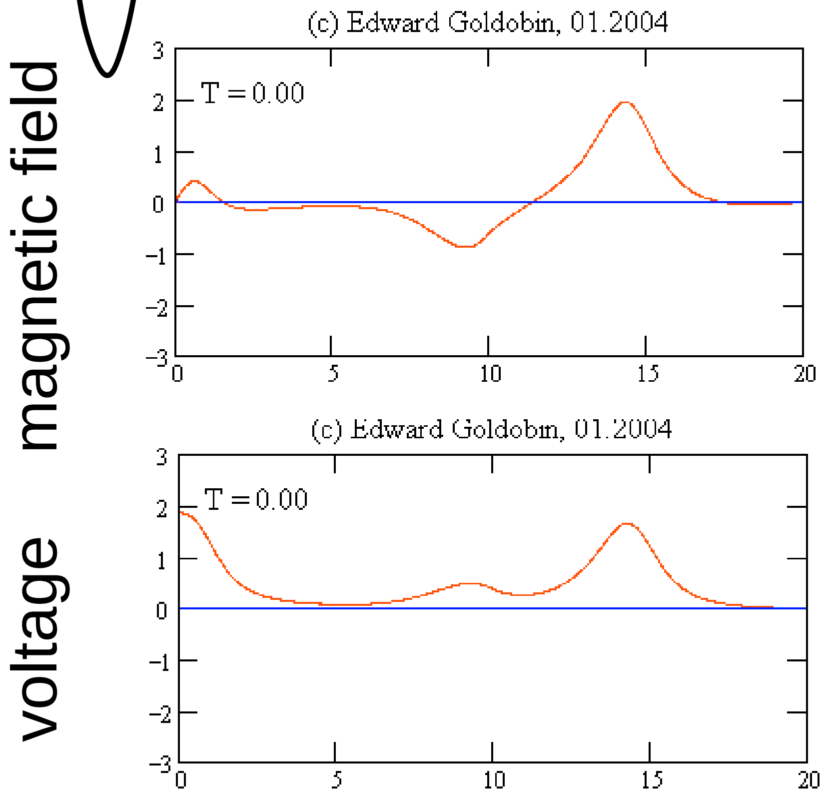


📖 Malomed et al., PRB **69**, 064502 (2004)
📖 Goldobin et al., PRB **70**, 094520 (2004)

Overcritical bias:oscillator





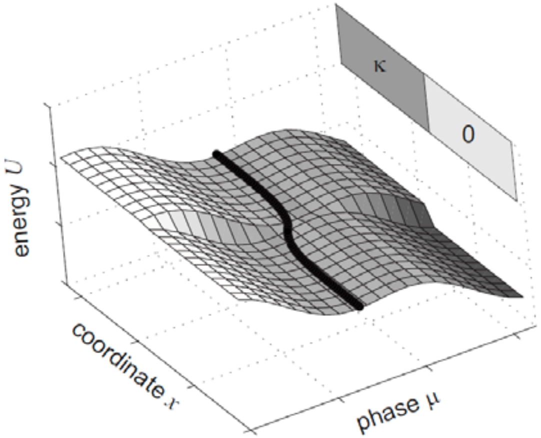


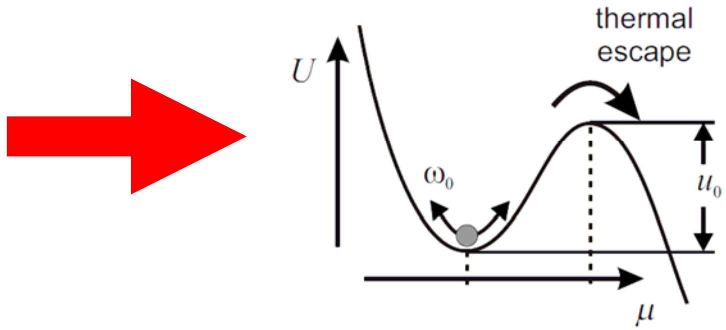
- Frequency depends on:
  - bias current, damping, length
  - two outputs shifted by 180°
  - more stable than flux-flow due to semifluxon pinning

N. Lazarides, PRB **69**, 212501 (2004).

Fractional vortex escape: theory

Theory: no theory for fractional vortex escape





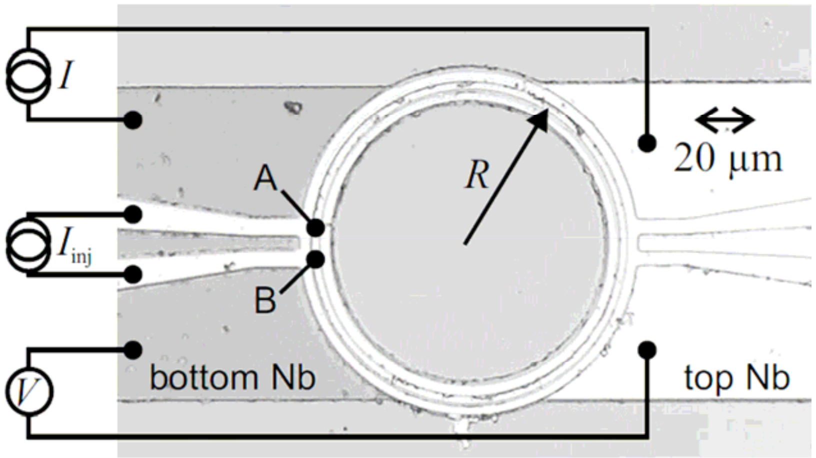
Mapping (theory for $L=\infty$):

- eigenmode expansion in the vicinity of depinning current
- single (lowest) eigenmode approximation. Not valid for $\kappa\to 0$!
- Applicable for both thermal and quantum escape

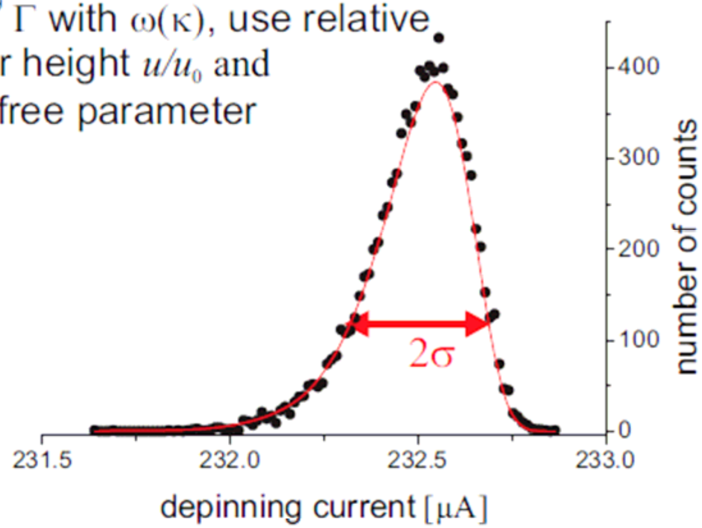
Vogel et al., Phys. Rev. B **80**, 134515 (2009)
Kato et al., J. Phys. Soc. Japan **66**, 1445 (1997)

Experiment

Sample: ALJJ

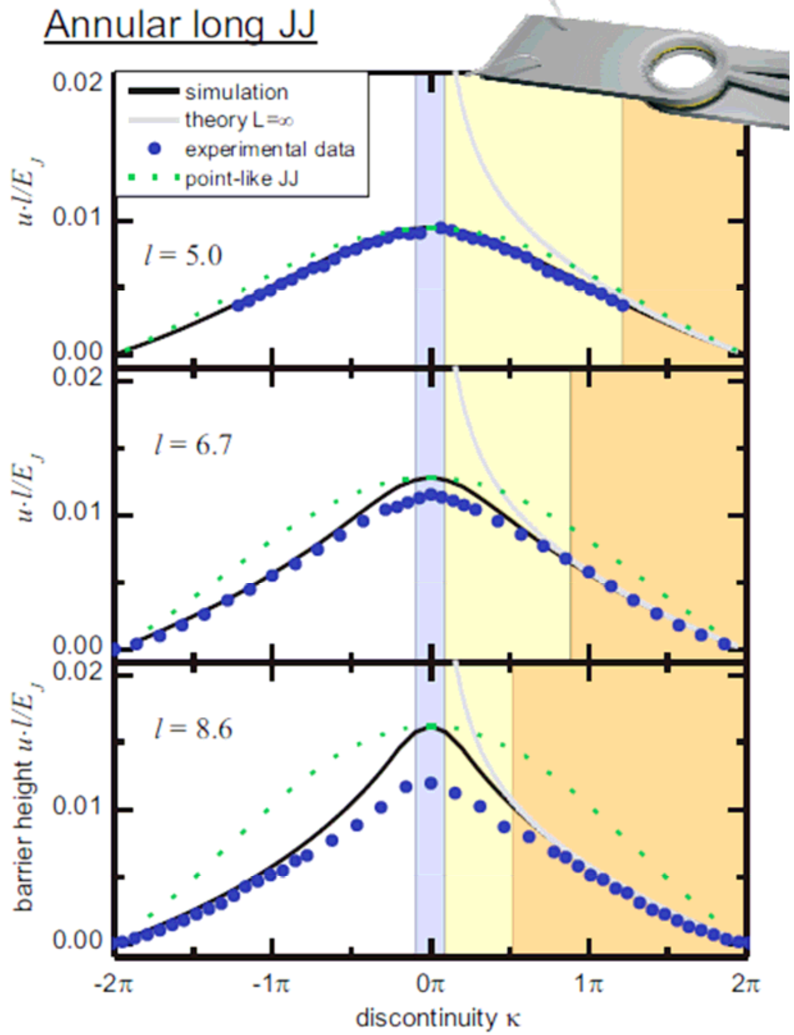


Measure depinning current distribution
⇒Fit[19] Γ with $\omega(\kappa)$, use relative barrier height u/u_0 and I_{c0} as free parameter



Vortex or flat phase escape?

Annular long JJ

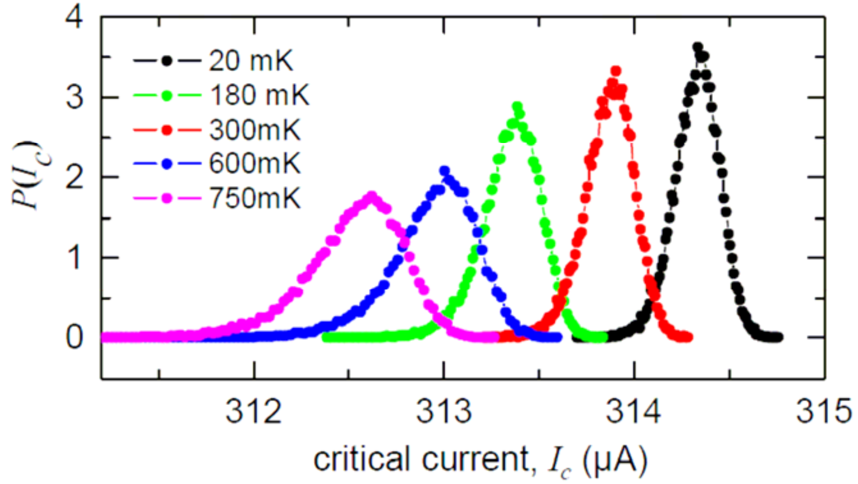


📖 Vogel et al., Phys. Rev. B **80**, 134515 (2009)

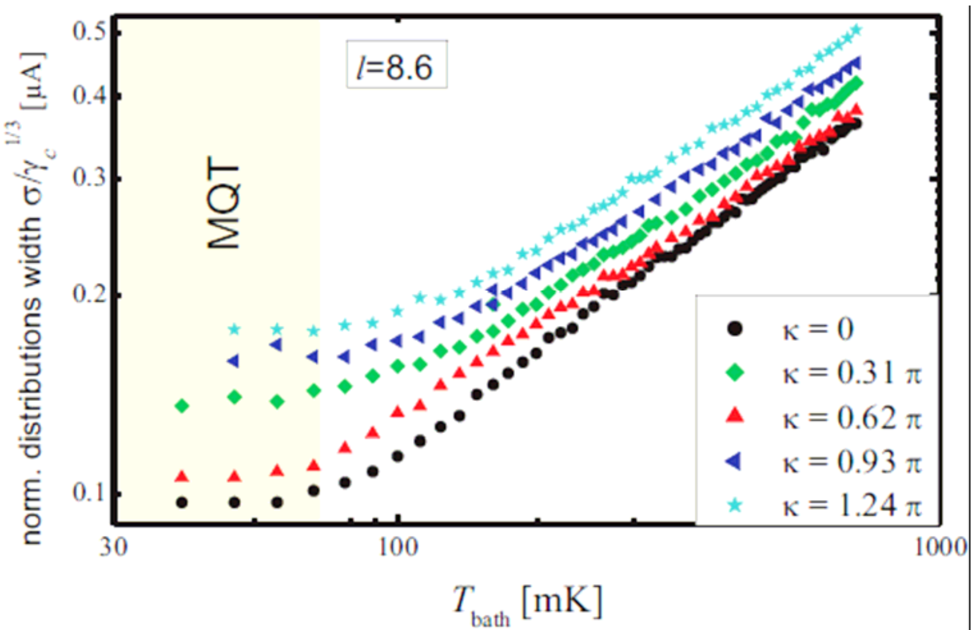
📖 Kienzle et al., PRB **80**, 014504 (2009)

Thermal escape vs. MQT

Below some T the width of escape histogram should saturate



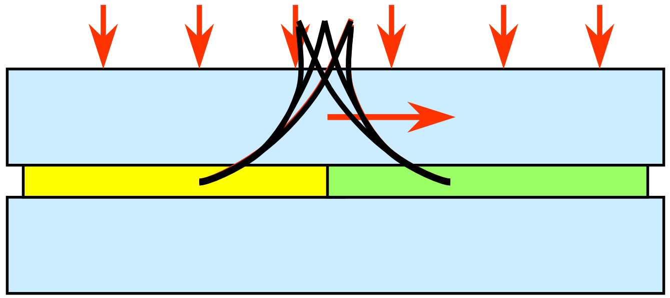
$$T^* = \frac{\hbar\omega_0}{2\pi k_B} \approx 80\ \mathrm{mK}$$

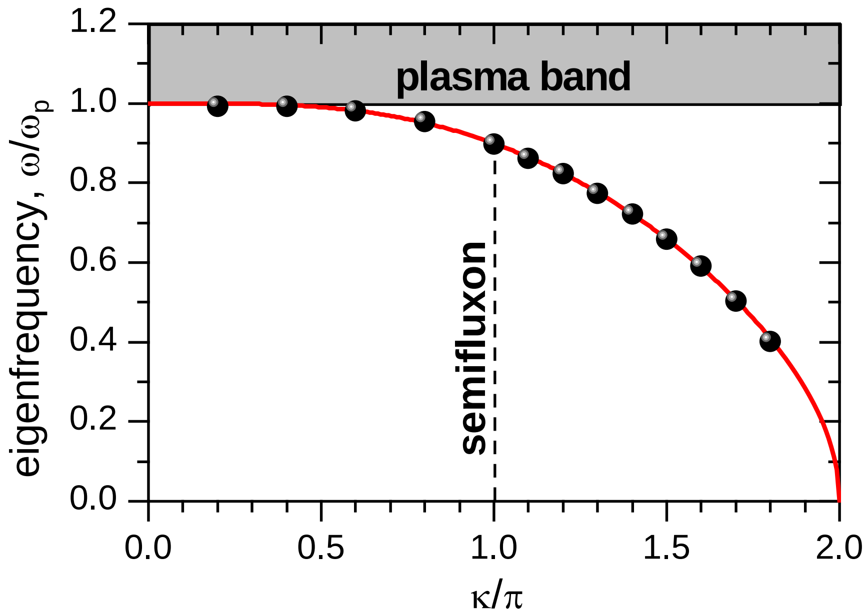


Possible observation of MQT of fractional vortices!

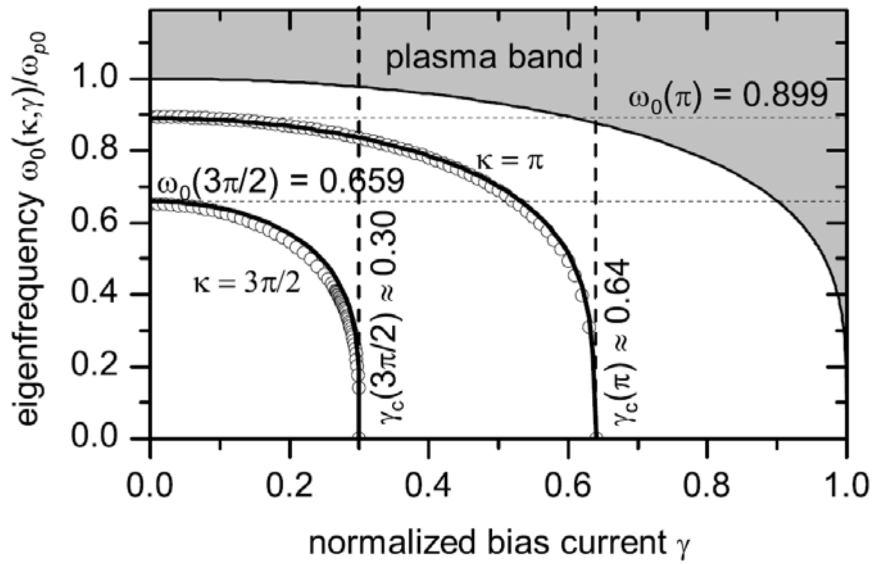
Eigenfrequency of a frac. vortex

- vortex is pinned at 0-κ boundary





$$\omega_0(\kappa) = \sqrt{\frac{1}{2}\cos\frac{\kappa}{4}\left(\cos\frac{\kappa}{4} + \sqrt{4 - 3\cos^2\frac{\kappa}{4}}\right)}$$



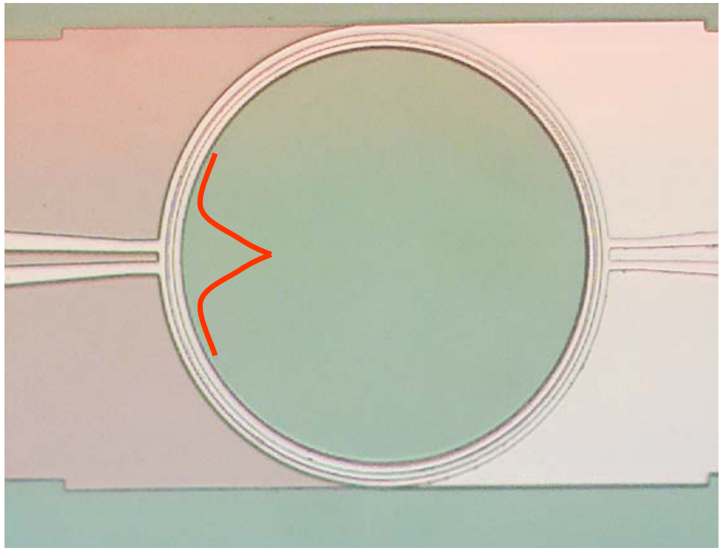
Important for:
- dynamics (parasitic resonances),
- stability analysis,
- quantum tunneling (prefactor),
- constructing tunable metamaterials.

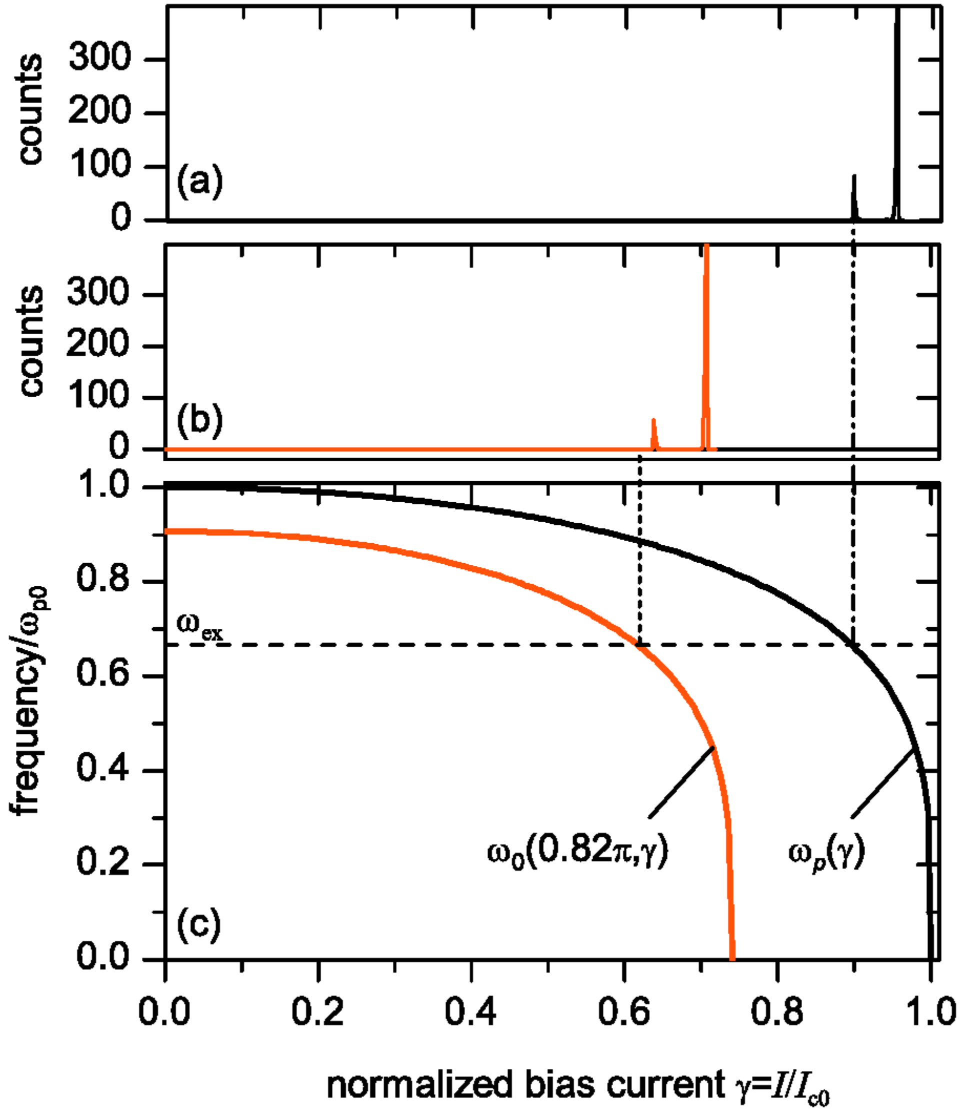
H. Susanto et al., PRB **71**, 104518 (2005)

Eigenfrequency spectroscopy

- $\omega_0(\kappa)$-dependence
- no flipping $\kappa \leftrightarrow 2\pi-\kappa$
- resonant excitation → underdamped LJJ

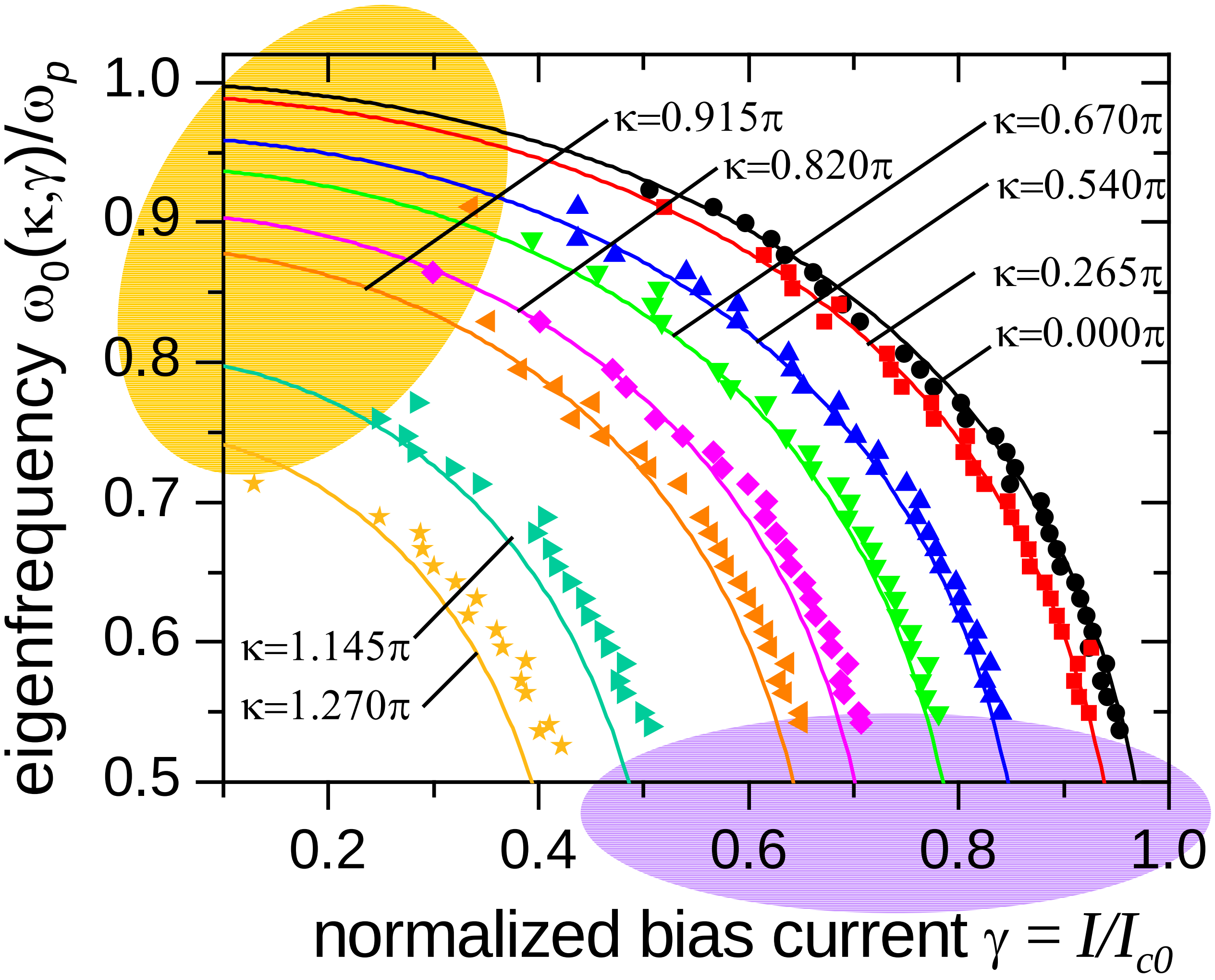
Nb-AlOx-Nb annular LJJ
with injectors!





K. Buckenmaier et al., Phys. Rev. Lett. **98**, 117006 (2007)

Spectroscopy Results



- small bias, deep well, large amplitudes, nonlinear resonance
- large bias, close indistinguishable maxima, prop. to noise

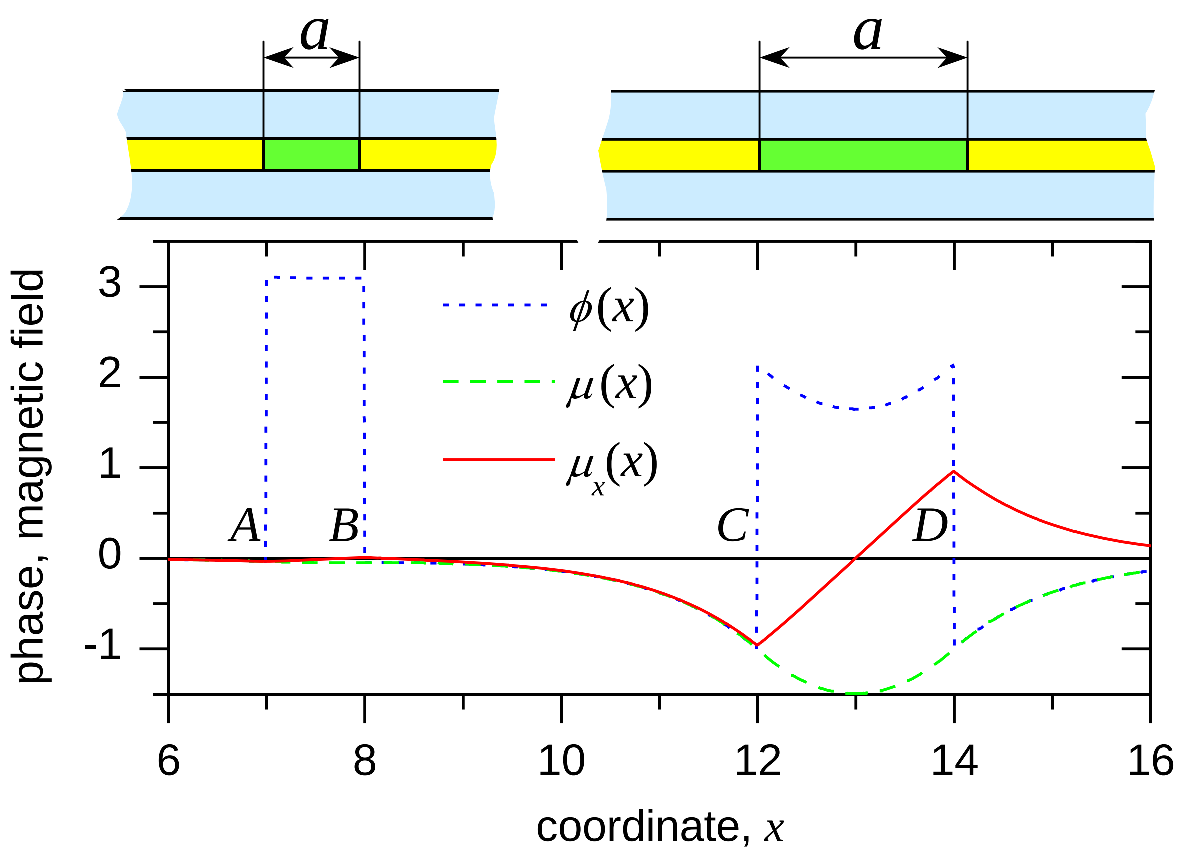
K. Buckenmaier et al., Phys. Rev. Lett. **98**, 117006 (2007)

Two coupled semifluxons

=

semifluxon molecule

π-facet of length a: ↑↓ vs. μ=0



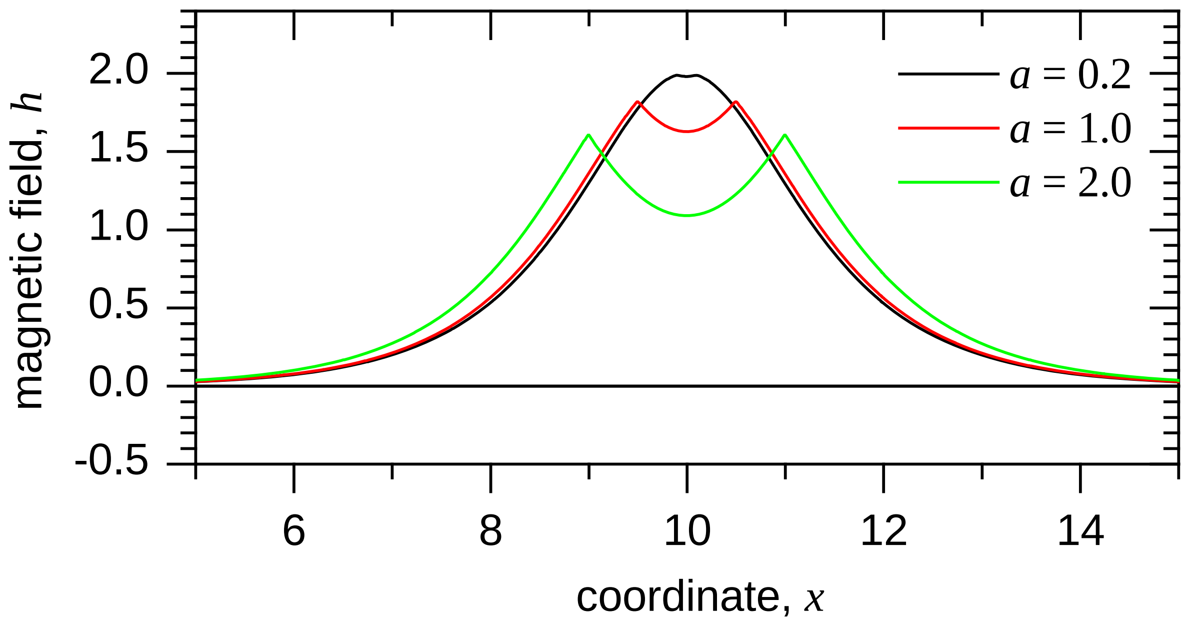
- Two 0-π-boundaries at a distance a:
  - semifluxons in ↑↓ state are formed for $a>a_c$
  - flux-less flat phase solution $(0–\pi–0)$ for $a<a_c$.

$a_c \sim \pi/2\ \lambda_J$

📖 Goldobin et al., Phys. Rev. B **67**, 224515 (2003)

Unipolar (FM) states

- semifluxon+semifluxon = fluxon!



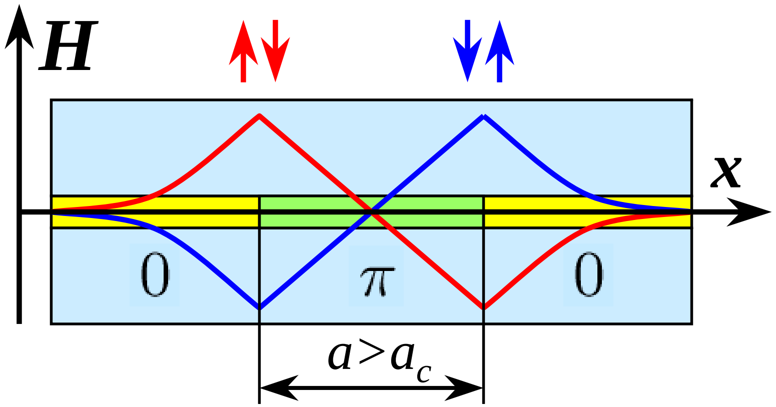
$$U_{\uparrow\uparrow}(\infty) = 2U_{SF} \approx 4.6, \quad U_{\uparrow\uparrow}(0) = U_F = 8$$
$$U_{\uparrow\downarrow}(\infty) = 2U_{SF} \approx 4.6, \quad U_{\uparrow\downarrow}(0) = 0$$

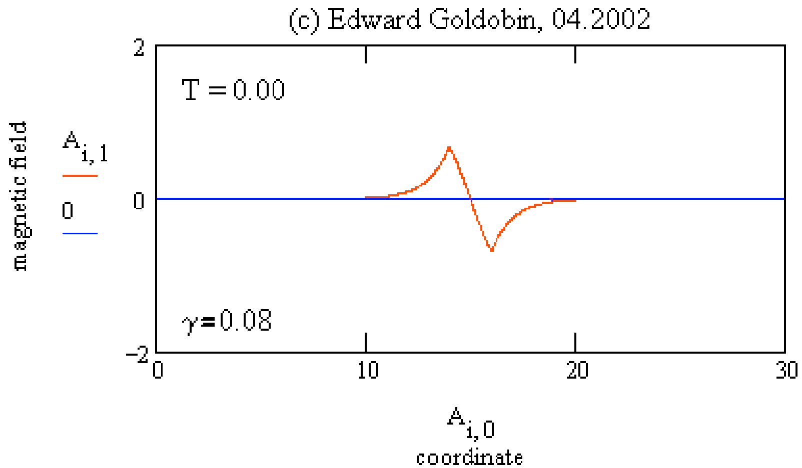
$$U_{\uparrow\uparrow}(a) \geq U_{\uparrow\downarrow}(a)$$

Two biased semifluxons

- The distance between 0-π boundaries $a > a_c$
- ↑↓ state at zero bias
- current pushes semifluxons to each other => swap

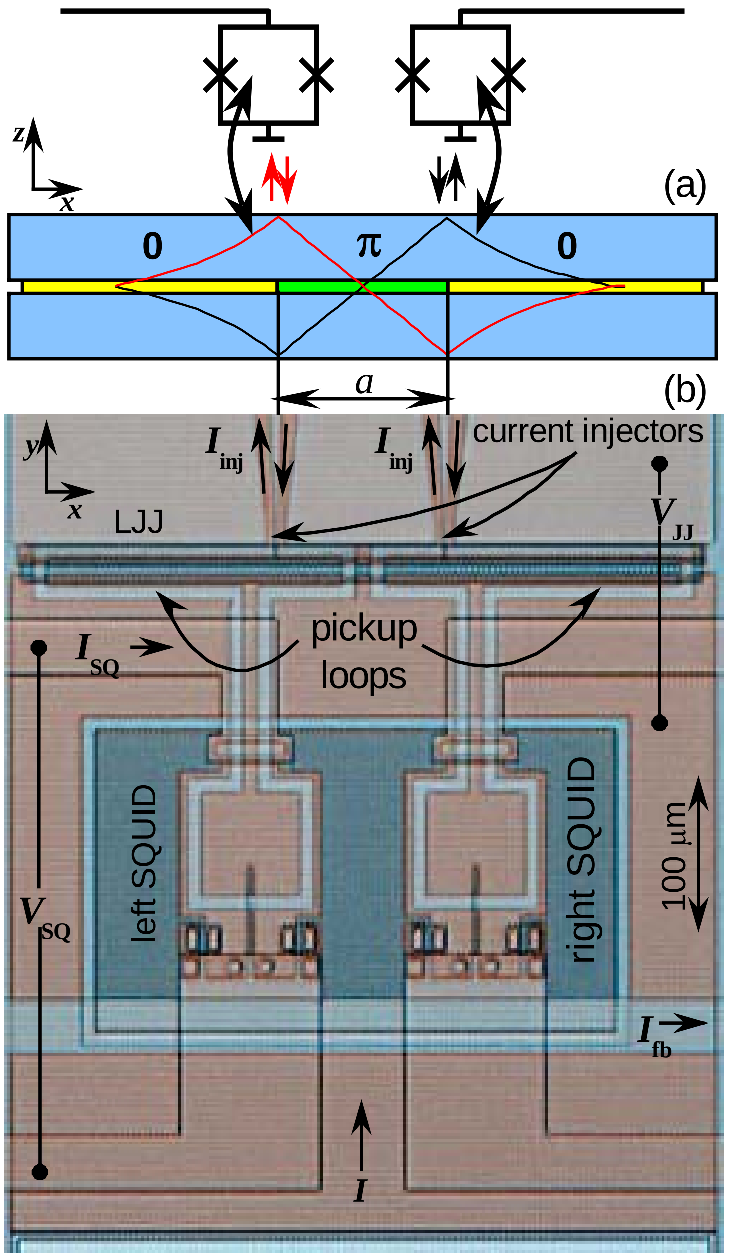
$$\uparrow\downarrow \xrightarrow{\gamma=0.08} \downarrow\uparrow$$

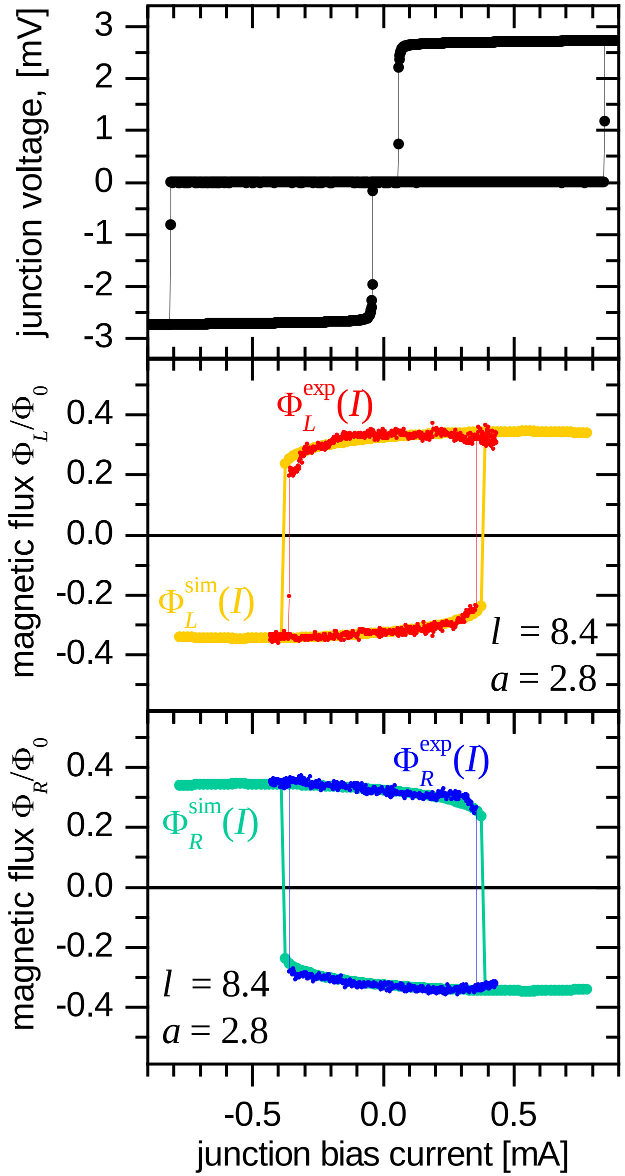




📖 Goldobin et al., PRB **67**, 224515 (2003)

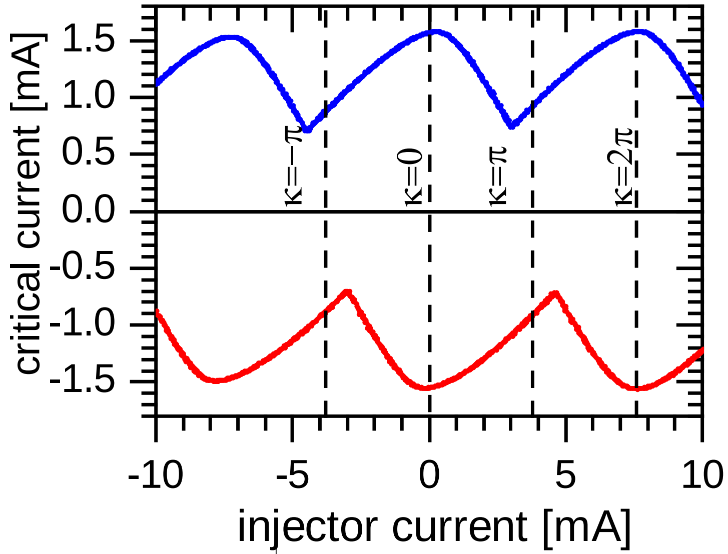
Observed rearrangement: ↑↓↔↓↑

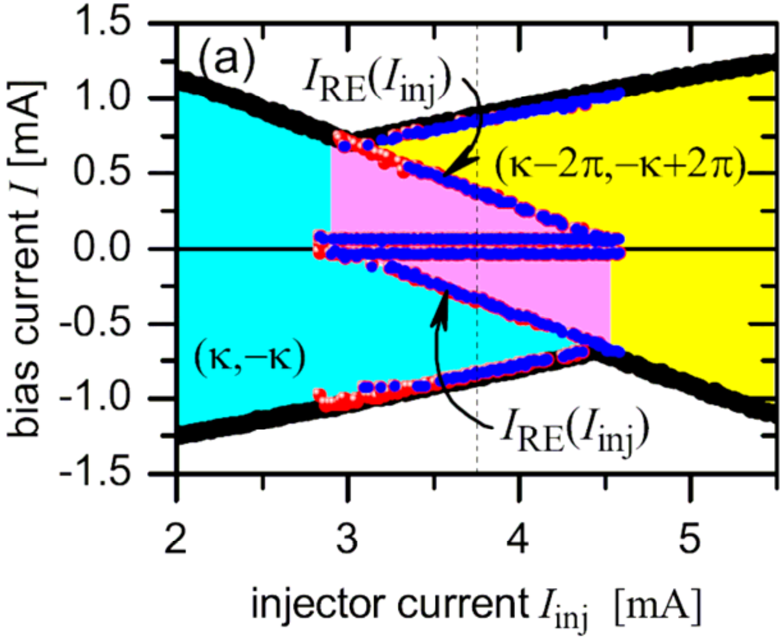


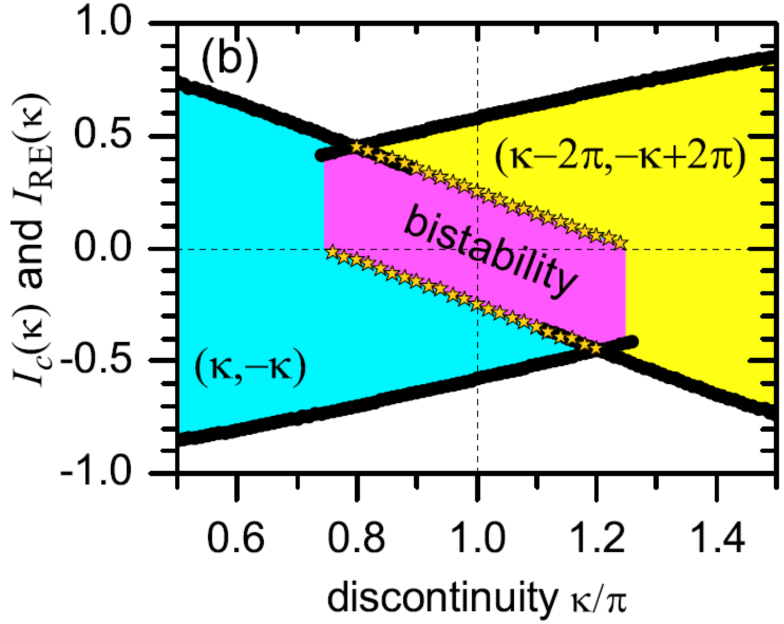


A. Dewes et al., PRL **101**, 247001, (2008)

Bistability region (κ)





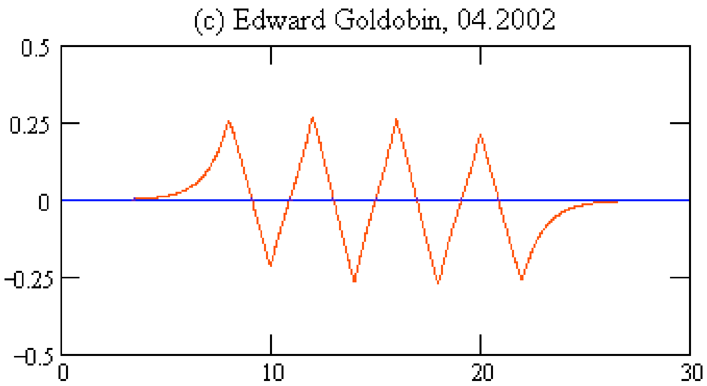


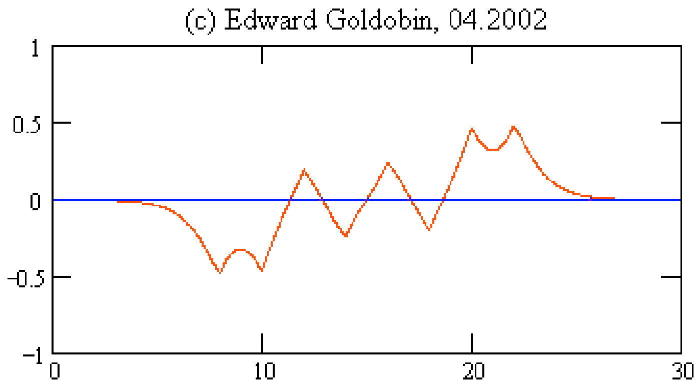
📖 A. Dewes et al., PRL **101**, 247001, (2008)

Rearranging 8 semifluxons

- The distance between corners $a > a_c$
- AFM ordered state at zero bias

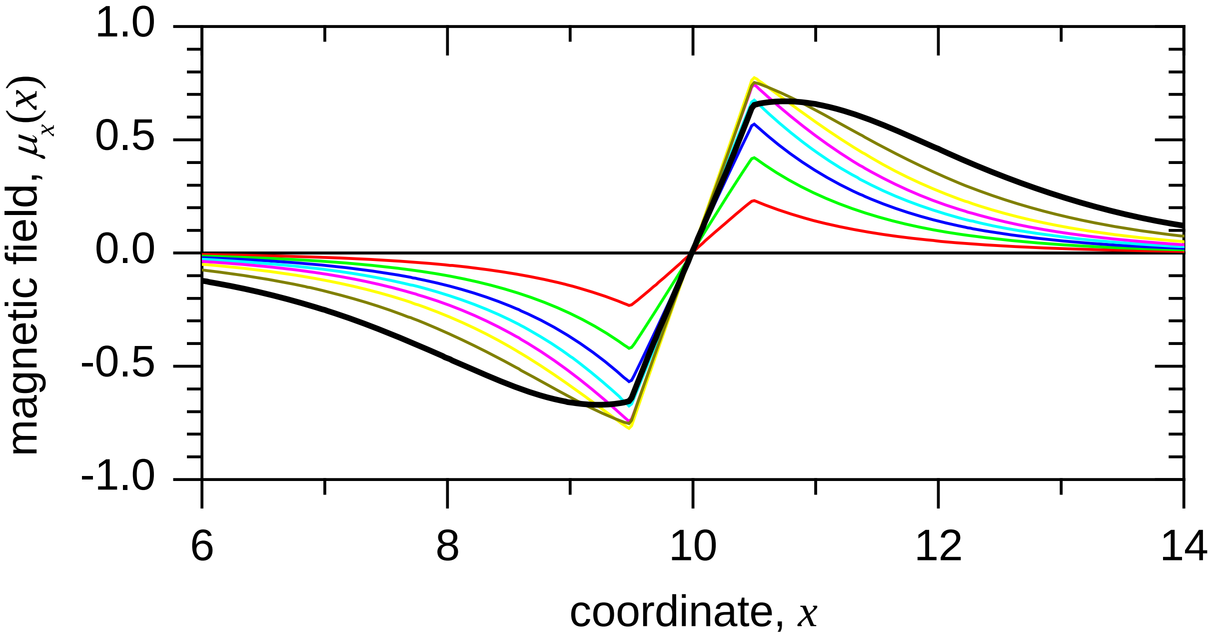
$$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow \xrightarrow{\gamma=+0.13} \downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow \xrightarrow{\gamma=+0.23} \downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow$$





π facet of length $a<a_c$ biased

- The distance between corners $a < a_c$
- flat phase state at zero bias
- increasing bias with step 0.1. $\gamma_c = 0.76$

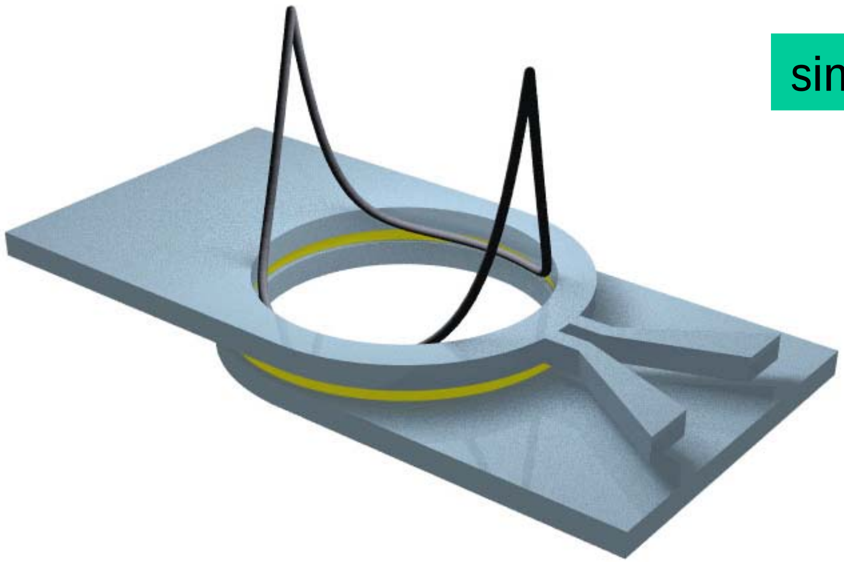


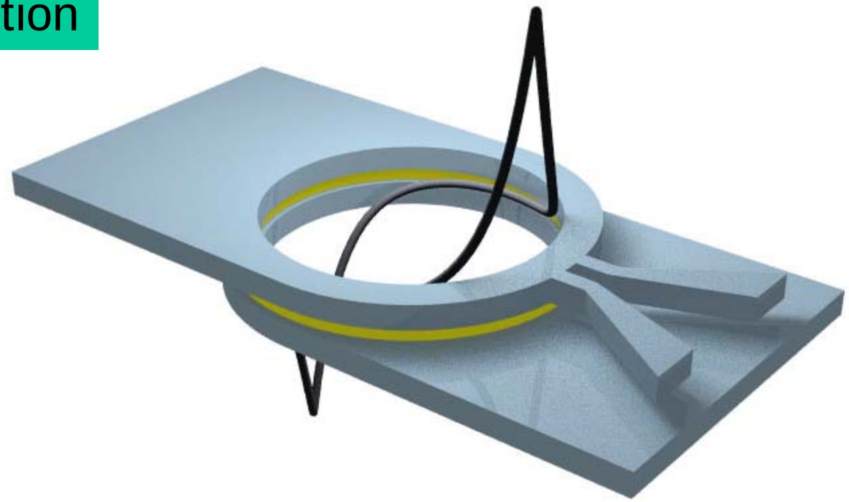
"semifluxons" emerge under the action of bias current

Goldobin et al., PRB **67**, 224515 (2003).

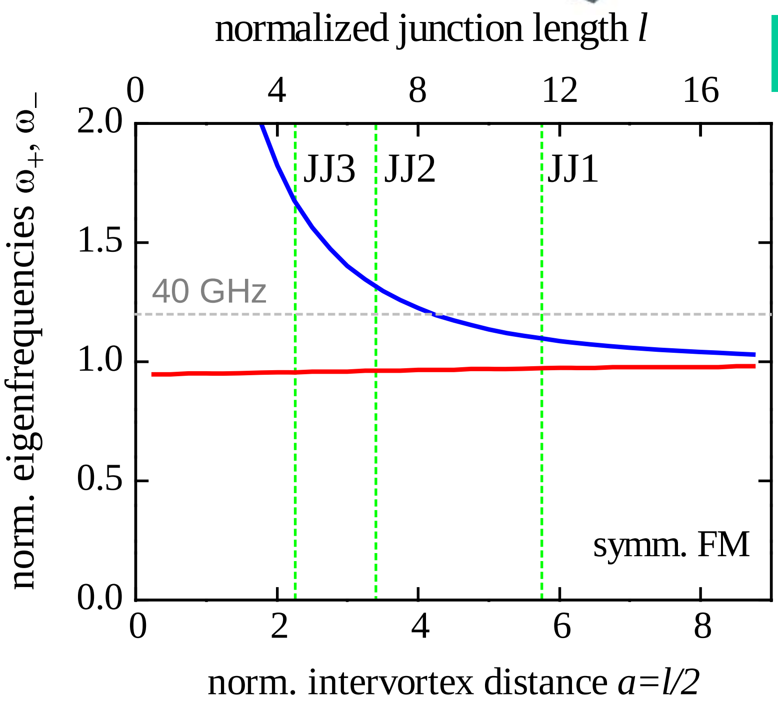
Splitting of eigenmodes

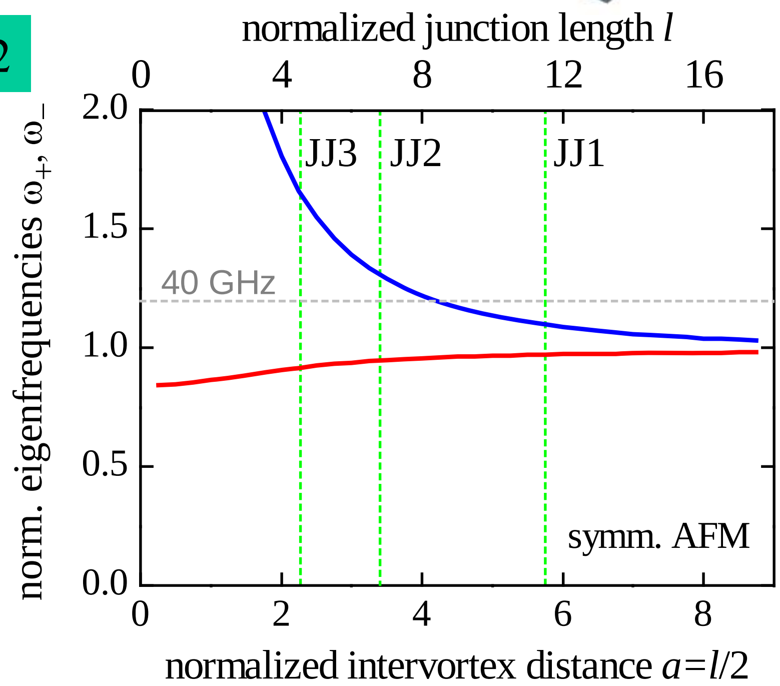
simulation



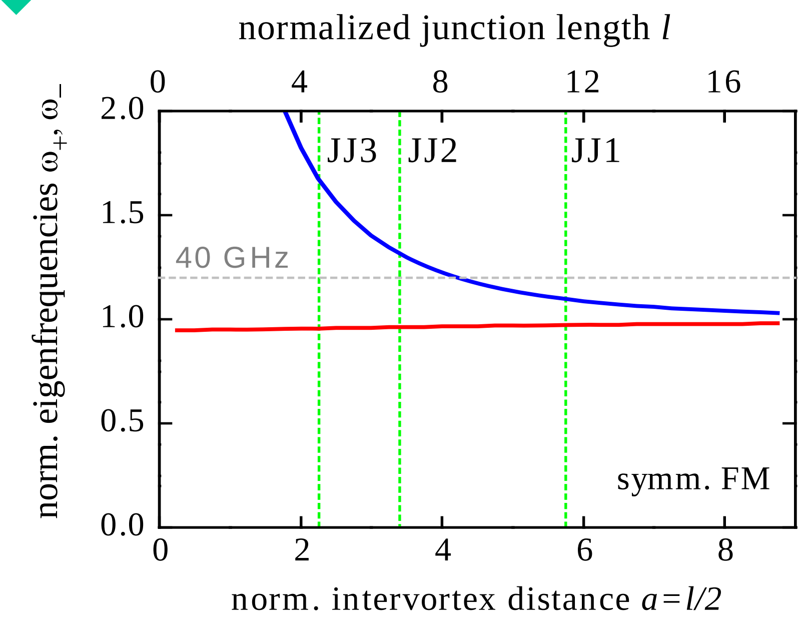


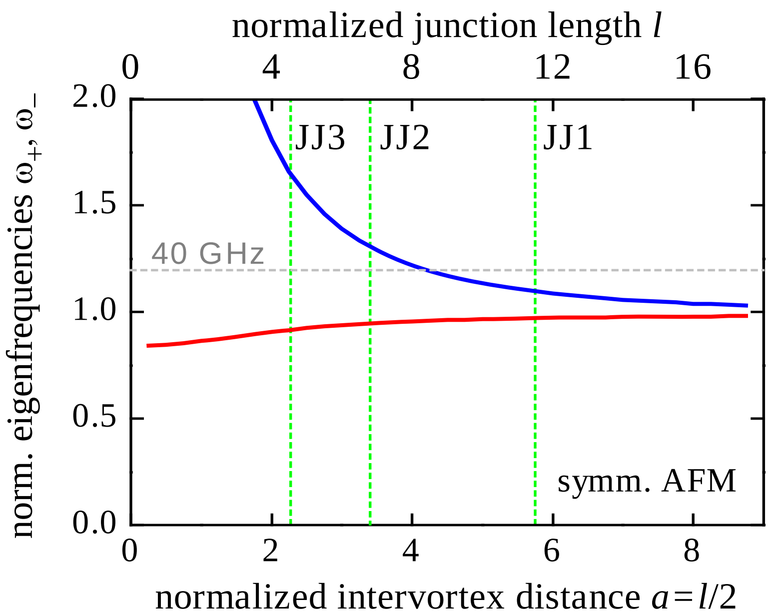
κ = π/2

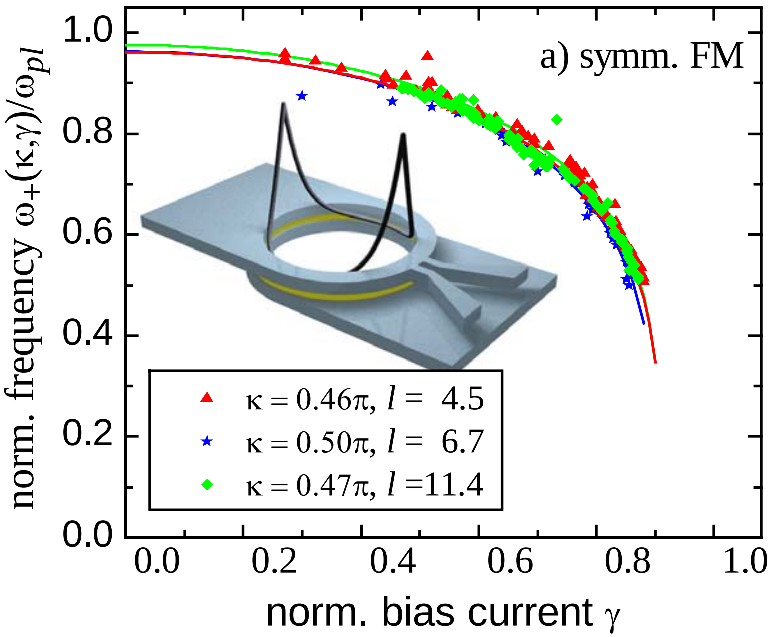


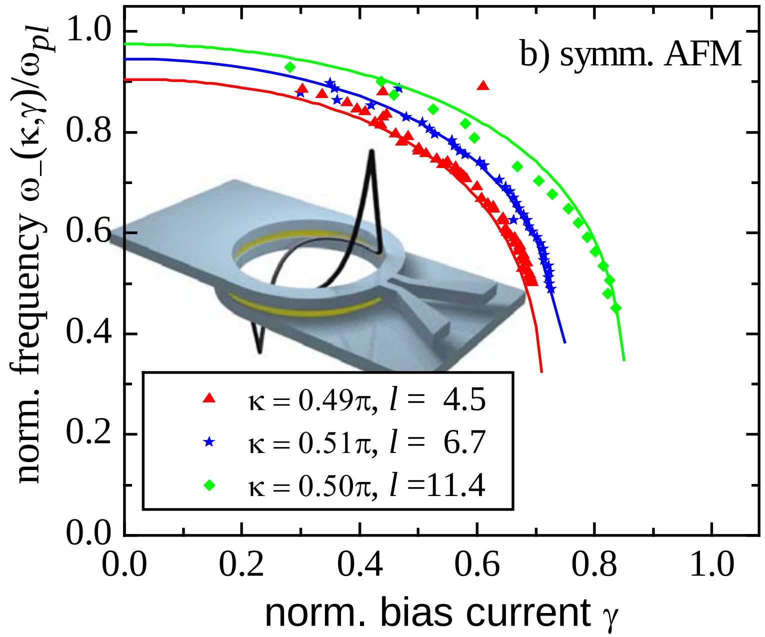


Lowest mode vs. distance



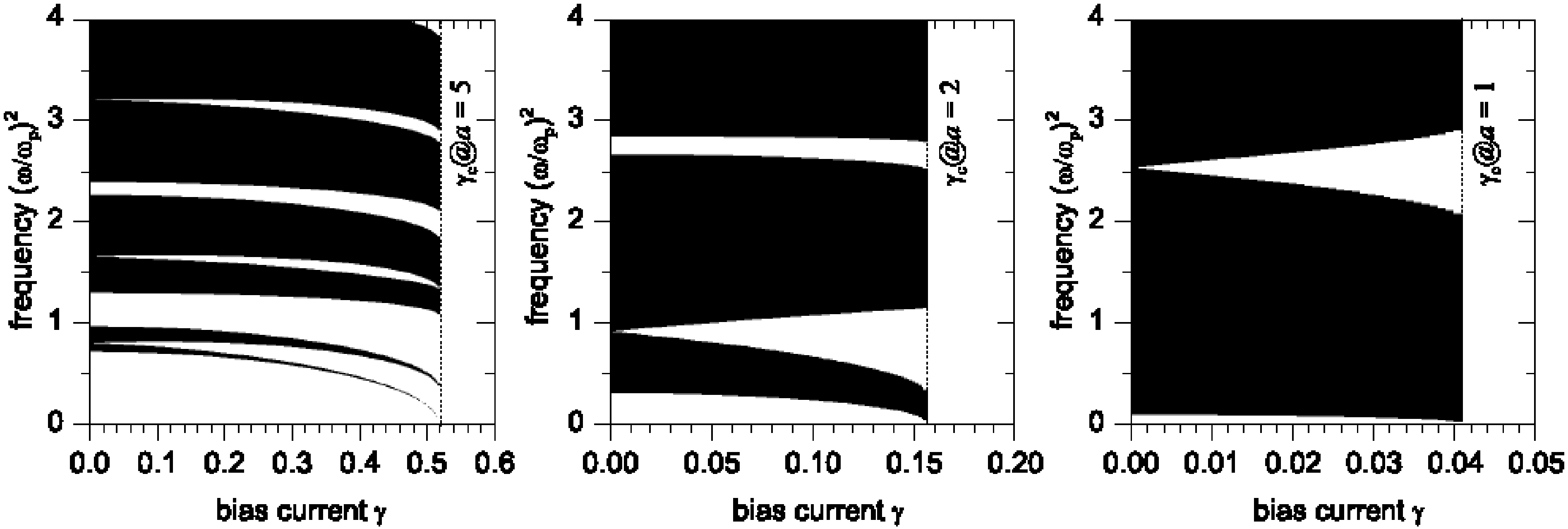


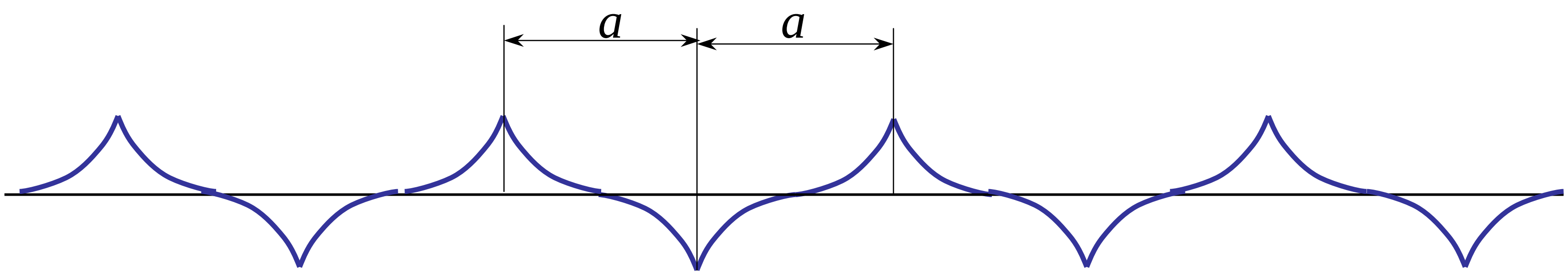




Tunable plasmonic crystal

Photonic (plasmonic) crystal of fractional vortices.
Photonic crystal tutorial: http://ab-initio.mit.edu/photons/tutorial/





H. Susanto, E. Goldobin, et al., PRB **71,** 174510 (2005)
see the **poster #16 of S. Buehler et al.** (KIT) during this meeting

Quantum fractional vortices?

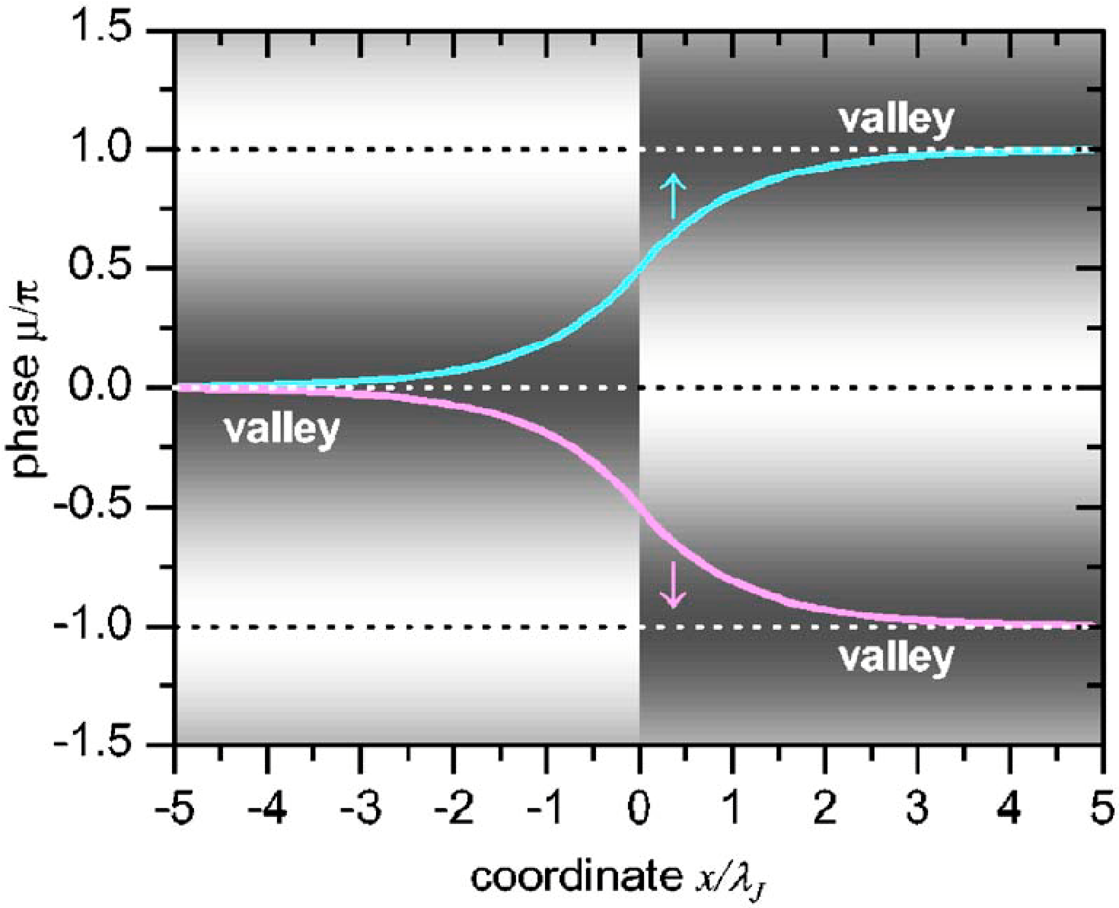
Aren’t we already quantum?

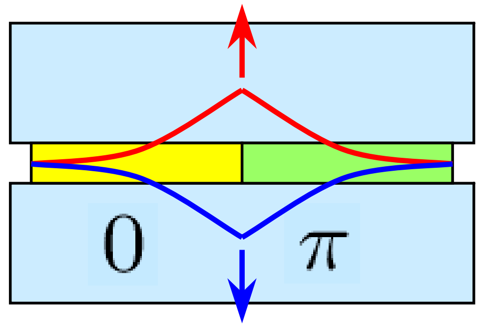
(Superconductivity, Josephson, half flux quantum)

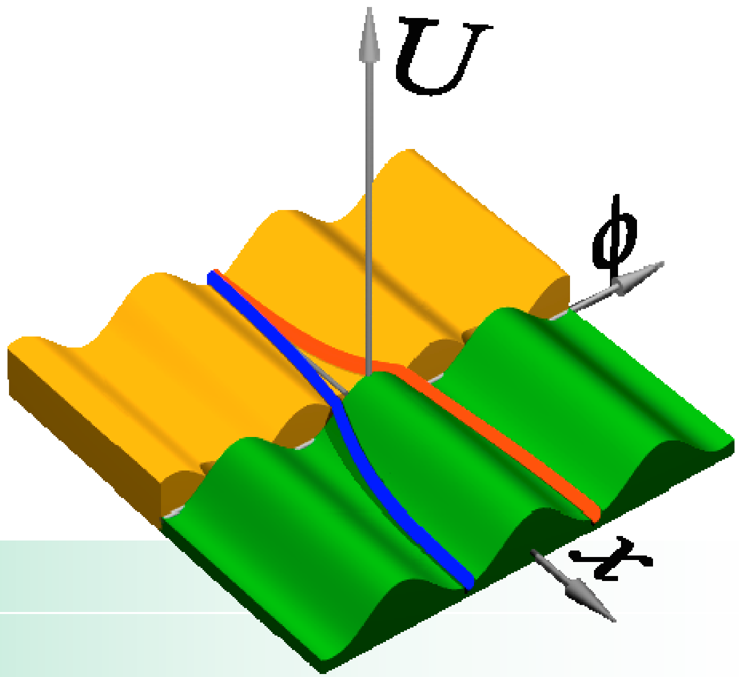
Can not quantized vortices be quantum?

Let’s jump into quantum realm ones again!

MQC with 1 semifluxon



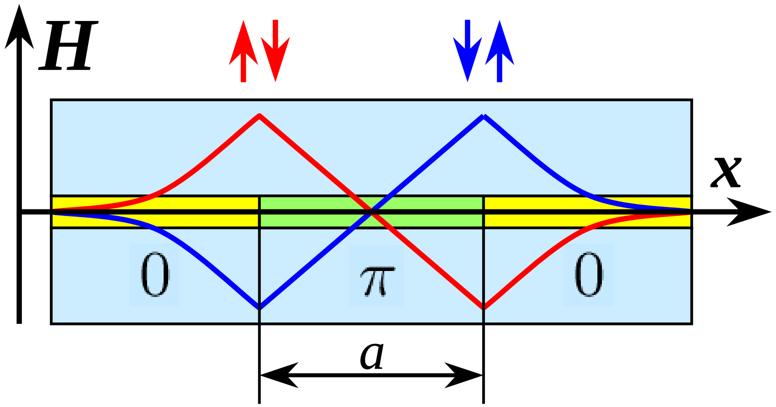


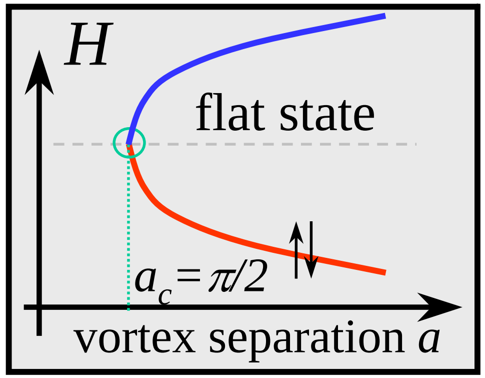


Long JJ: 🕮 E. Goldobin et al., PRB **72**, 054527 (2005)

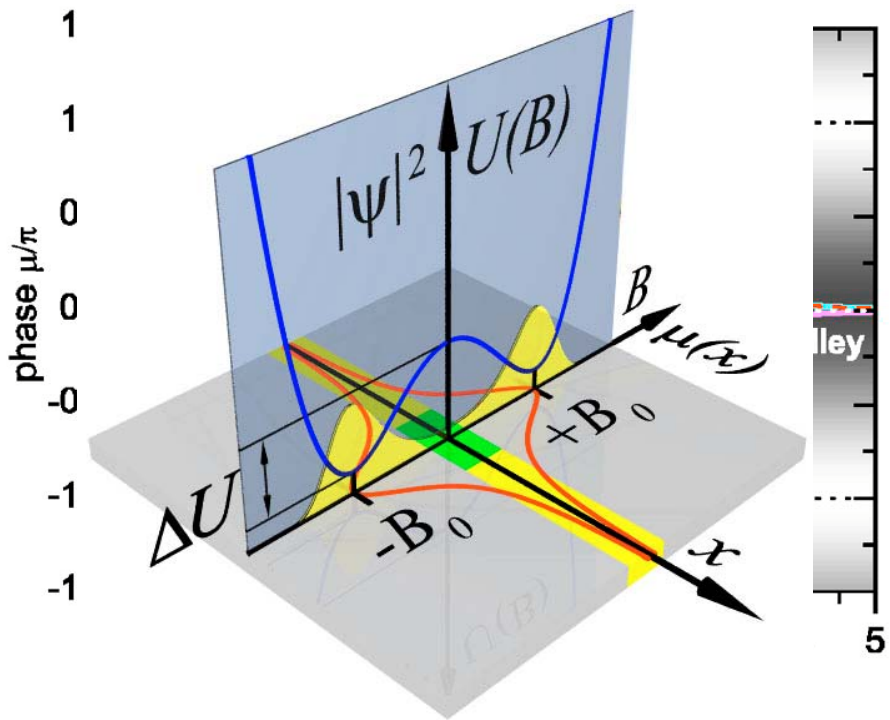
Short JJ: 🕮 E. Goldobin et al., PRB **81**, 054514 (2009)

MQC in two-vortex molecule





$$a = a_c + \delta a$$



(w=1μm, j_c=100A/cm^2,λ_L=100nm)

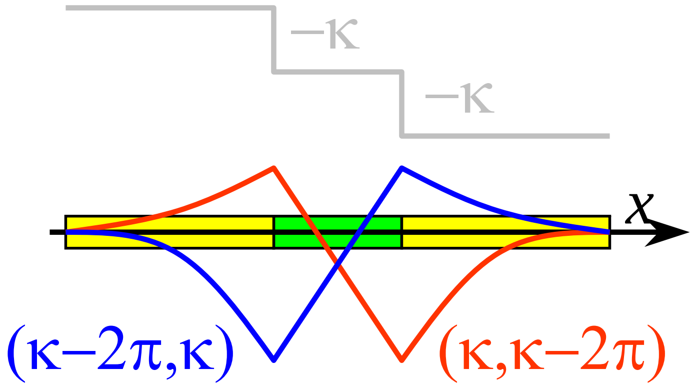
quantum effects start to play a role for $\delta\tilde{a} \lesssim 0.02$.

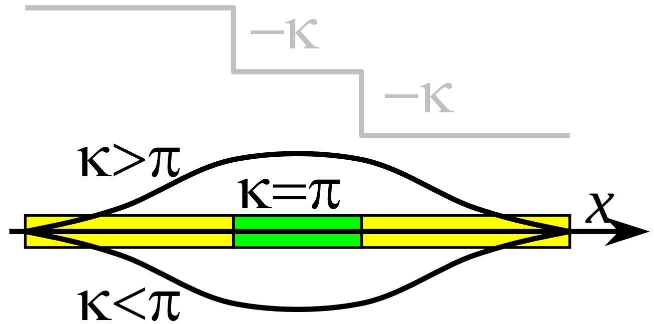
$$T^\star = \frac{\Delta U}{k_B} = \frac{E_J \lambda_J}{k_B} \frac{8}{\pi + 2} \delta\tilde{a}^2 .$$

For $\delta\tilde{a} = 0.01$, we obtain $T^\star \approx 130$ mK

📖 Goldobin et al., PRB **72**, 054527 (2005)

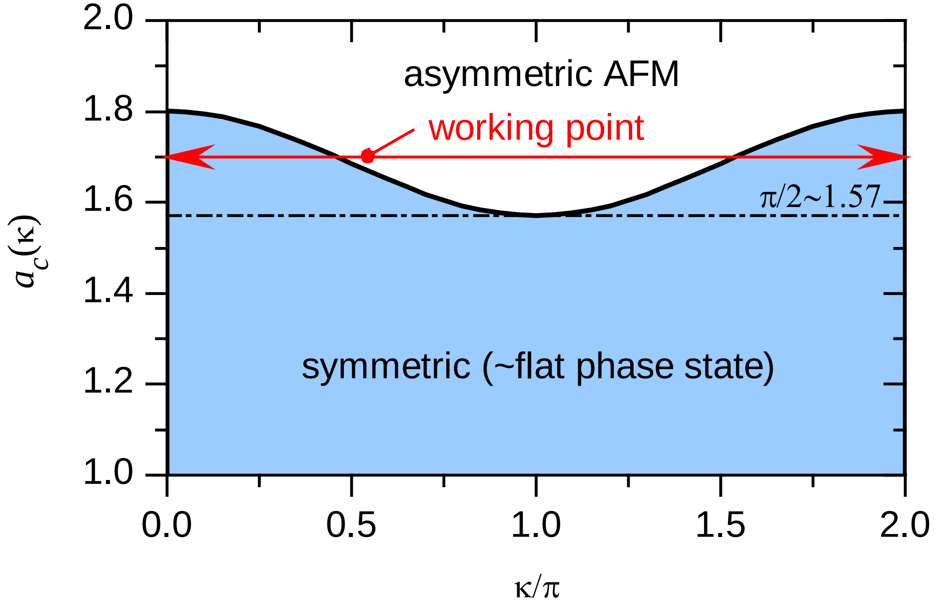
Tunable barrier for AFM molecule

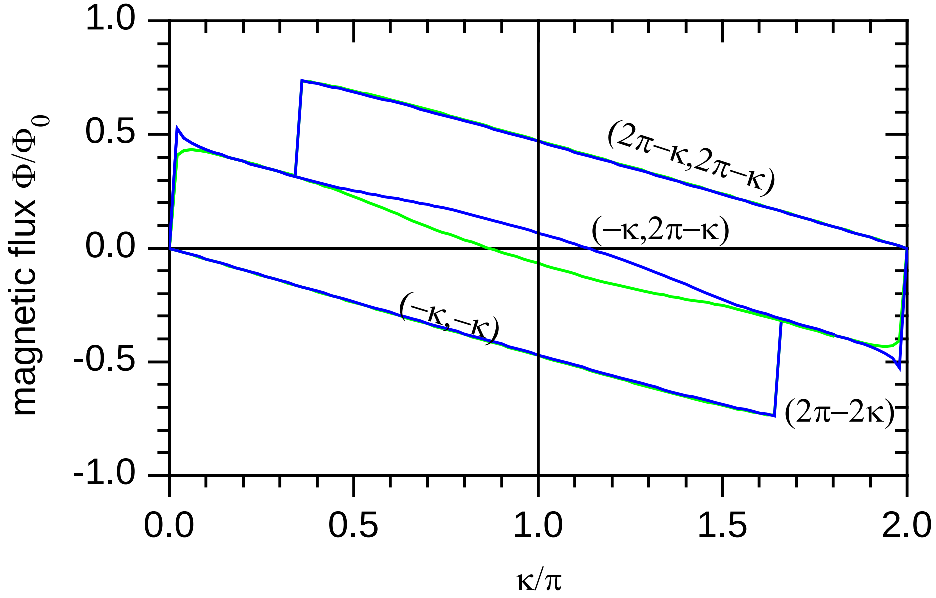




2 degenerate states

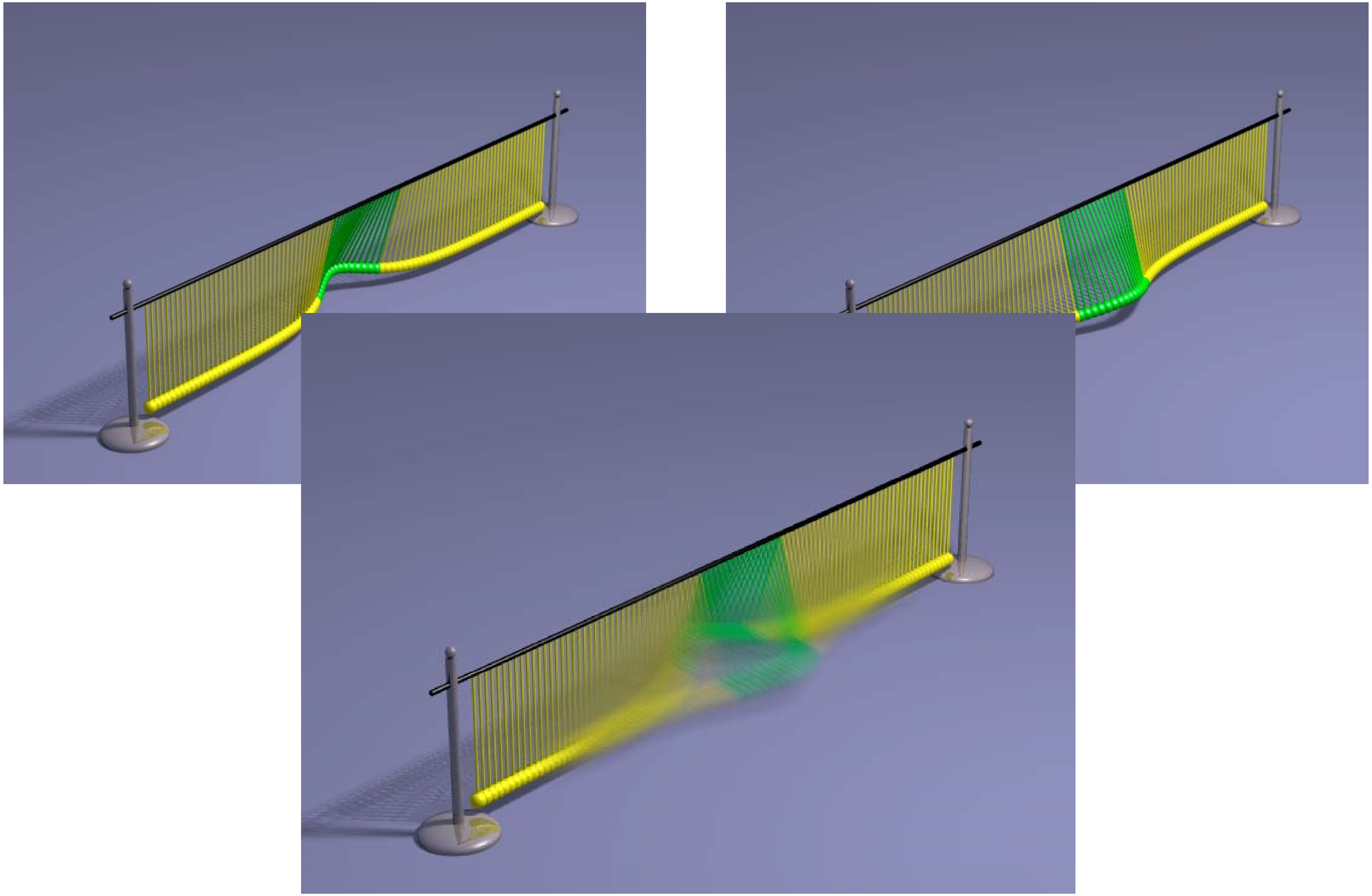
1 state (~ flat phase state)





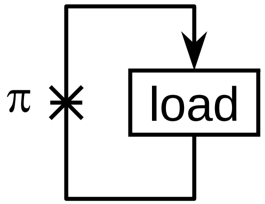
D. Heim et al. ArXiv: 1208.6176

Quantum pendula chains

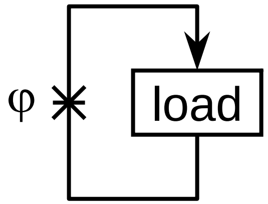


φ Josephson junction

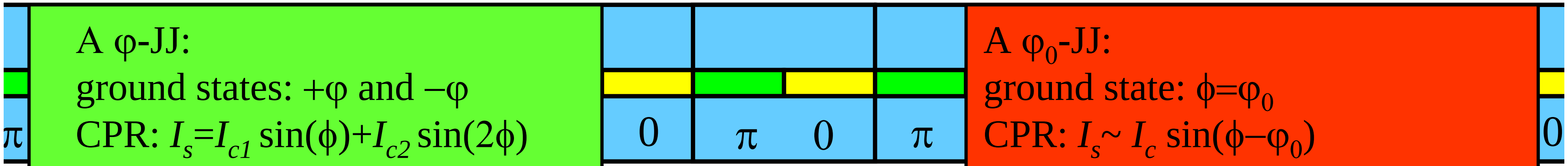
- π-Josephson junctions are great…but φ-junction are even better ;-)



phase battery
(not dischargable)
[1]



Proposal of Mints, Buzdin and co. [2]



A φ-JJ:
ground states: +φ and −φ
CPR: $I_s = I_{c1}\sin(\phi) + I_{c2}\sin(2\phi)$
$\varphi = \arcsin(-I_{c1}/2I_{c2})$, $I_{c2} < -I_{c1}/2$

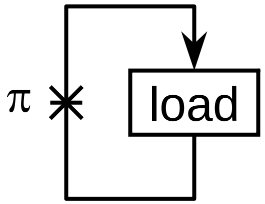
A φ_0-JJ:
ground state: $\phi = \varphi_0$
CPR: $I_s \sim I_c \sin(\phi - \varphi_0)$

How to understand that we have a φ JJ?
- close it in a loop :-(
- measure two I_{c+} and I_{c-} [3]
- measure $I_c(H)$ → How does it look like for φ JJ?

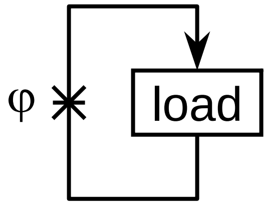
[1] T. Ortlepp et al., Science **312**, 1495 (2006); A. Feofanov et al., Nat. Phys. **6**, 593 (2010)
[2] R.M. Mints et al. PRB **57**, R3221 (1998); A. Buzdin et al. PRB **67**, R220504 (2003).
[3] E. Goldobin et al. PRB **76**, 224523 (2007).

φ Josephson junction

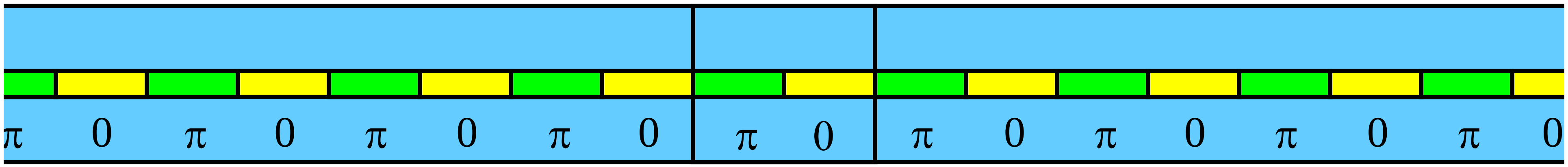
- π-Josephson junctions are great…but φ-junction are even better ;-)



phase battery
(not dischargable)
[1]



Proposal of Mints, Buzdin and co. [2]



What shall we measure to understand that we have obtained a φ JJ?
- close it in a loop and measure spontaneous current/magnetic flux!
- But, no way to characterize before hand

[1] T. Ortlepp et al., Science **312**, 1495 (2006); A. Feofanov et al., Nat. Phys. **6**, 593 (2010)
[2] R.M. Mints et al. PRB **57**, R3221 (1998); A. Buzdin et al. PRB **67**, R220504 (2003).

φ Josephson junction

Definition: φ-JJ [2]

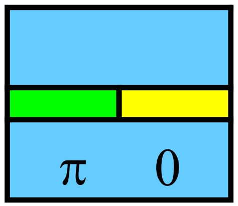
ground states: $\phi = +\varphi$ and $\phi = -\varphi$
CPR: e.g. $I_s = I_{c1}\sin(\phi) + I_{c2}\sin(2\phi)$
$\varphi = \arcsin(-I_{c1}/2I_{c2})$, $I_{c2} < -I_{c1}/2$

Definition: φ_0-JJ [1]

ground state: $\phi = \varphi_0$
CPR: $I_s \sim I_c \sin(\phi - \varphi_0)$

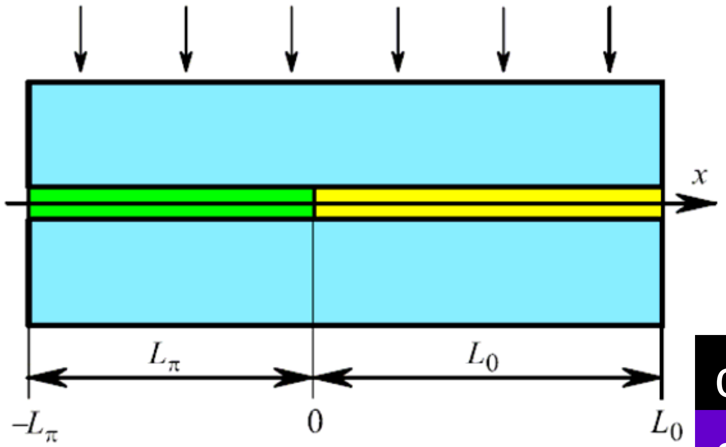
What shall we measure to understand that we have a φ JJ?

- measure two I_{c+} and I_{c-} corresponding to escape from $+\varphi$ and $-\varphi$ [3]
- measure $I_c(H) \to$ How does it look like for φ JJ?



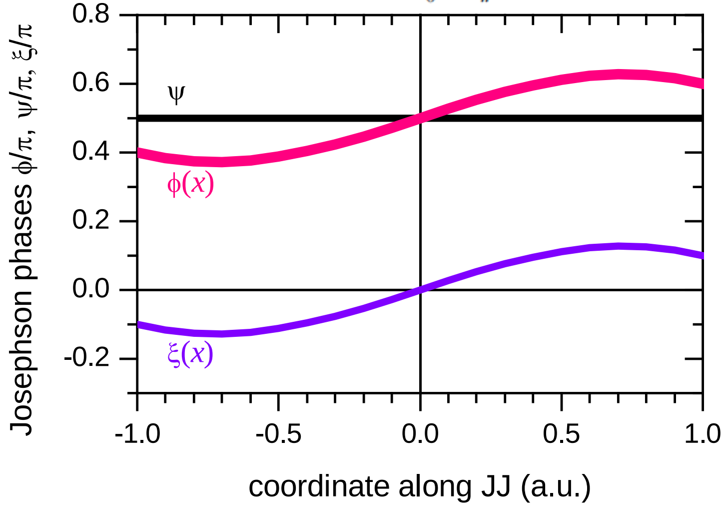
[1] A. Buzdin, PRL 101, 107005 (2008).
[2] A. Buzdin et al. PRB **67**, R220504 (2003).
[3] E. Goldobin et al. PRB **76**, 224523 (2007).

The main idea & result



$$j_c(x) = \langle j_c\rangle[1 + g(x)],$$

$$g(x) = \begin{cases} g_\pi = \frac{-2L_0}{L_0 - L_\pi}, & \text{for } x < 0; \\ g_0 = \frac{+2L_\pi}{L_0 - L_\pi}, & \text{for } x > 0, \end{cases}$$



$$\phi''(x) - j_c(x)\sin[\phi(x)] = -\gamma,$$

$$\phi(x) = \psi + \xi(x)\sin\psi, \quad \Big| \quad |\xi(x)\sin\psi| \ll 1$$

Taylor:

$$\xi''\sin\psi - \langle j_c\rangle[1 + g(x)][1 + \xi(x)\cos\psi]\sin\psi = -\gamma.$$

cons

$$\gamma = \langle j_c\rangle[\sin\psi + \langle g(x)\xi(x)\rangle\sin\psi\cos\psi]. \quad (11)$$

dev:

$$\xi'' - j_c(x)\cos\psi\,\xi(x) = \langle j_c\rangle[g(x) - \langle g\xi\rangle\cos\psi], \quad (12)$$

$$\xi'' = \langle j_c\rangle g(x).$$

$$\xi_\pi'(-L_\pi)\sin\psi = h; \qquad \xi_0'(L_0)\sin\psi = h,$$

$$\langle g\xi\rangle = \Gamma_0 + \Gamma_h\frac{h}{\sin\psi},$$

$$\Gamma_0 = -\frac{4}{3}\frac{L_0^2L_\pi^2}{L_0^2 - L_\pi^2}; \qquad \Gamma_h = \frac{L_0L_\pi}{L_0 - L_\pi}.$$

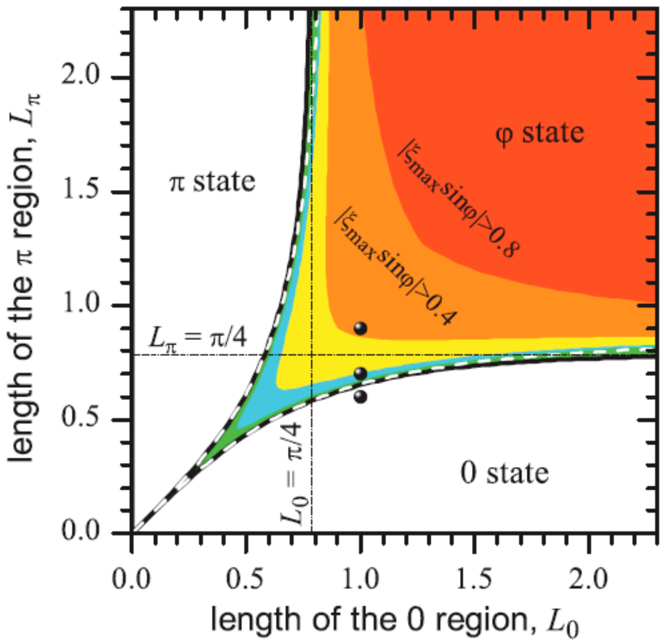
$$\gamma = \langle j_c\rangle\left[\sin\psi + \Gamma_h h\cos\psi + \frac{\Gamma_0}{2}\sin(2\psi)\right].$$

E. Goldobin et al., PRL **107**, 227001 (2011).

Phase diagram

phase diagram

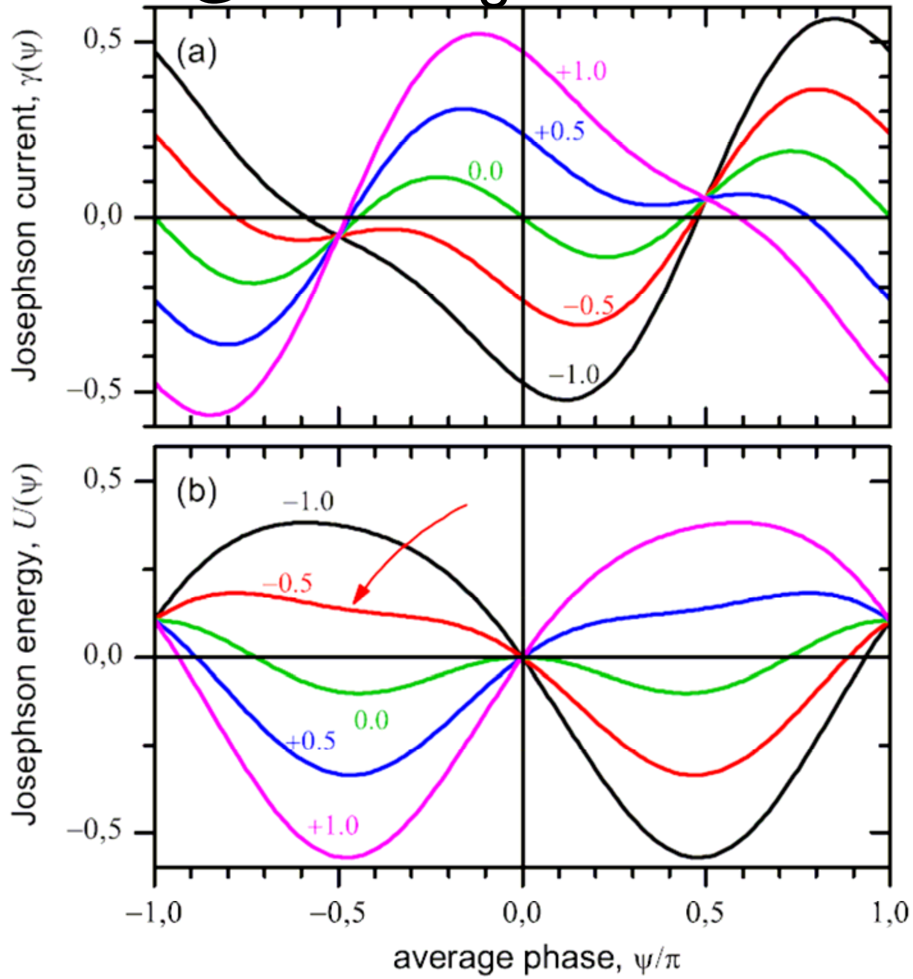
$$L_\pi \geq L_0\sqrt{\frac{3}{4L_0^2+3}}; \qquad L_0 \geq L_\pi\sqrt{\frac{3}{4L_\pi^2+3}}.$$



📖 Bulaevskii et al., JETP **25**, 290 (1977):

$$L_\pi \geq \arctan[\tanh(L_0)];$$
$$L_0 \geq \arctan[\tanh(L_\pi)].$$

@ diff. magnetic fields



$$U(\psi) = \langle j_c\rangle\left[1 - \cos\psi + \Gamma_h h \sin\psi + \frac{\Gamma_0}{2}\sin^2\psi\right].$$

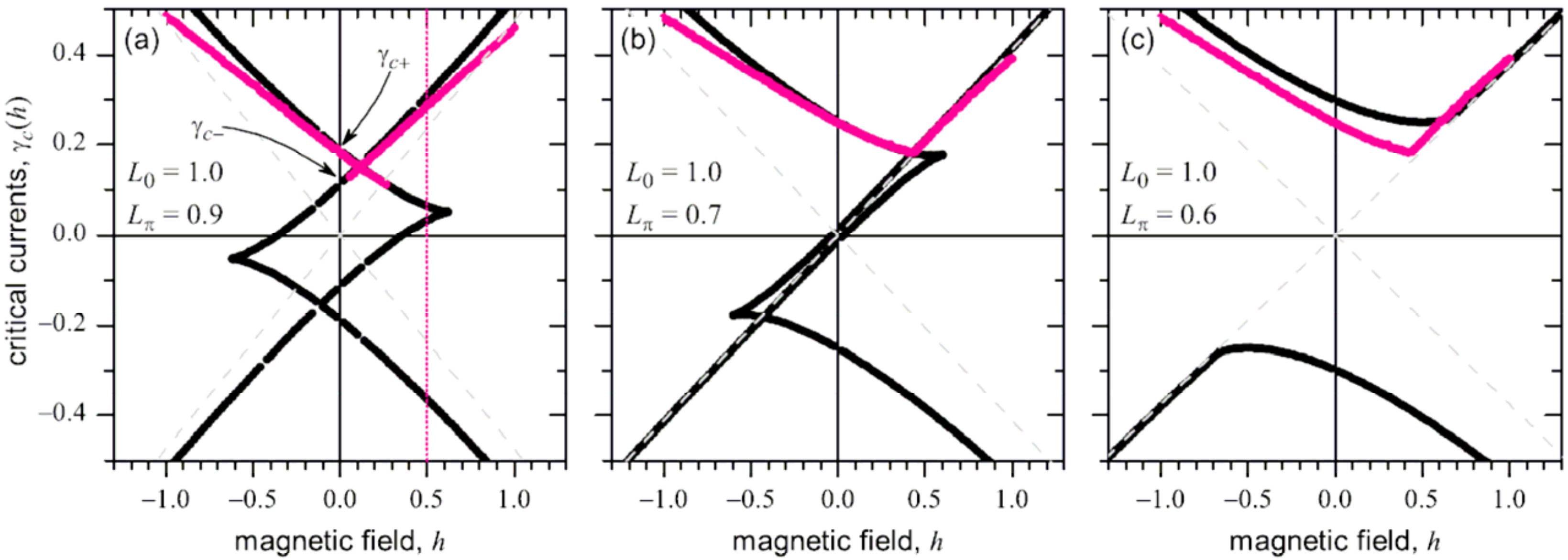
📖 E. Goldobin et al., PRL **107**, 227001 (2011).

Ic(H) dependence

$$\max_{\psi} \gamma(\psi) : d\gamma(\psi)/d\psi = 0$$

$$\cos\psi - \Gamma_h h \sin\psi - \Gamma_0(2\cos^2\psi - 1) = 0.$$

can be reduced to 4th order polynomial. **<u>All</u>** roots can be found numerically!



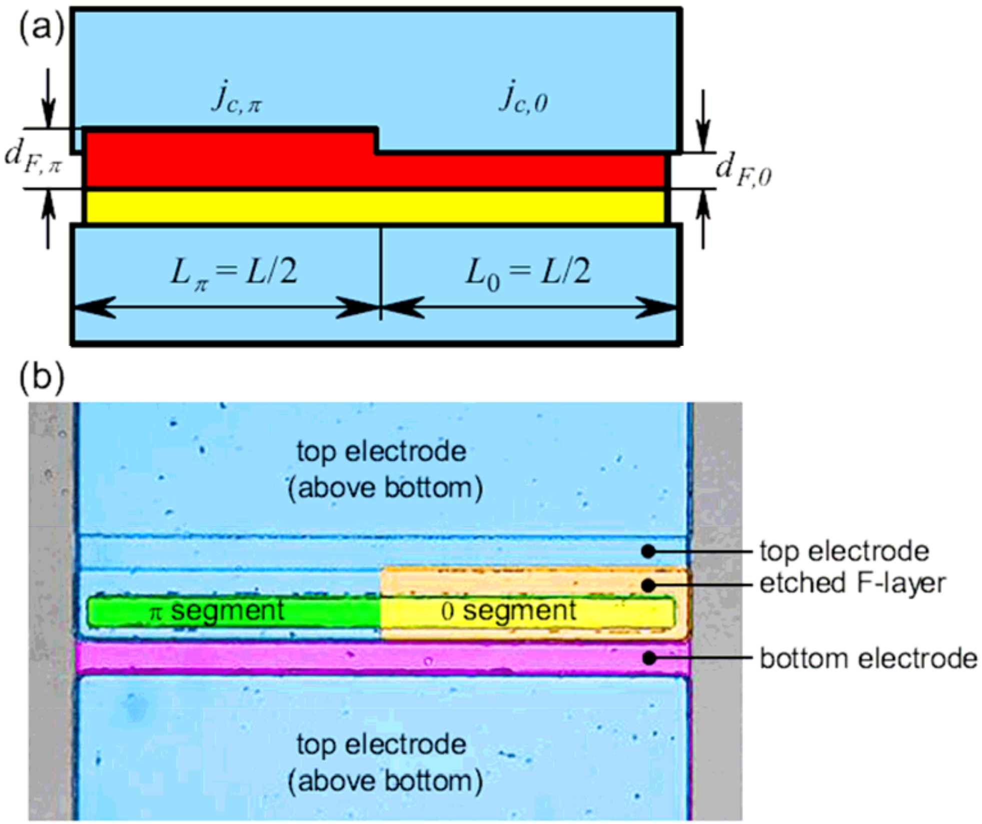
Rotated diamond-like figure:

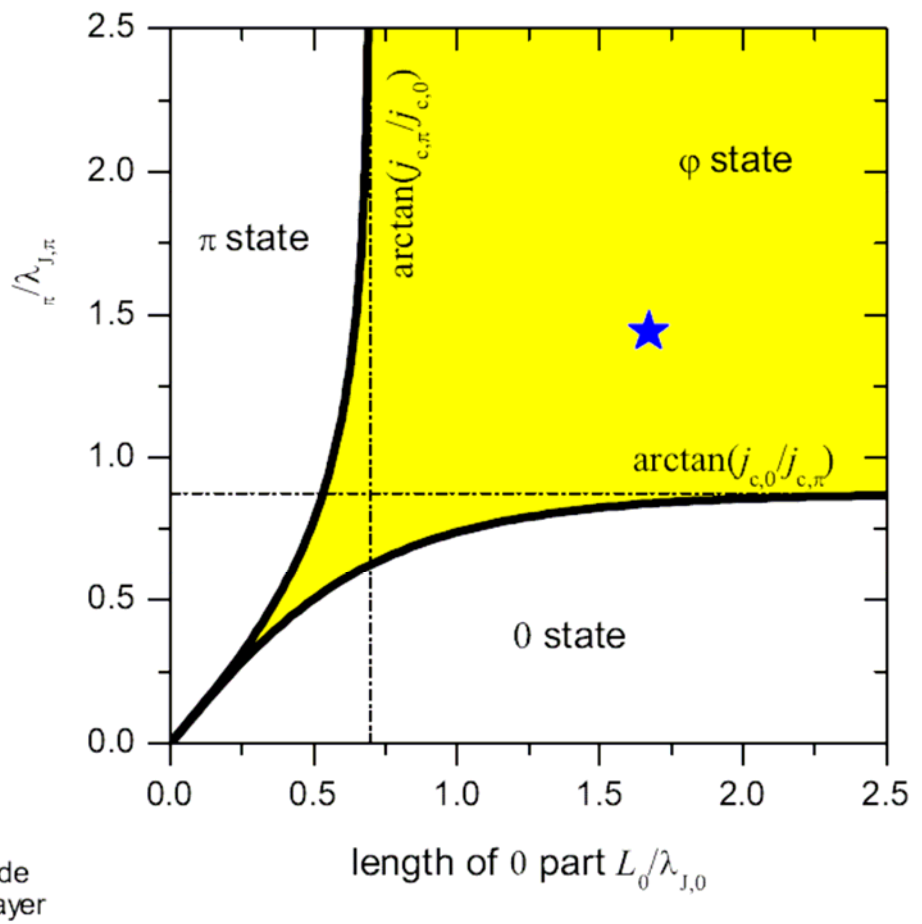
- 4 critical currents
- branches meet (local min. disappears)

$$\gamma_c^{as}(h) \approx \pm\langle j_c\rangle\Gamma_h h,$$

Experiment: samples

SIFS 0-π Josephson junction:
L = 100+100 μm,
j_{c0}=67.8μA/cm², $j_{c\pi}$=47.4A/cm²,
$L_0 \sim 1.73\ \lambda_{J,0}$, $L_\pi \sim 1.45\ \lambda_{J,\pi}$, @300mK

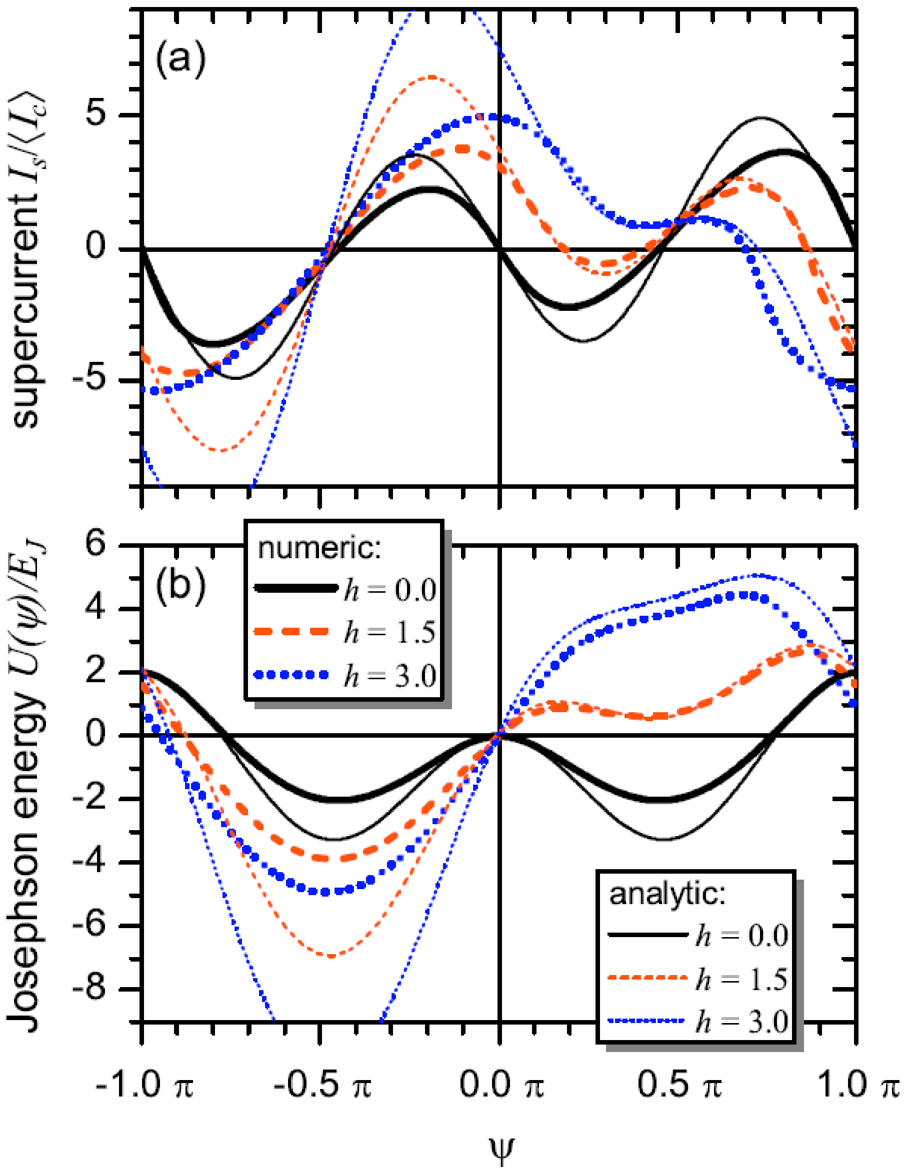


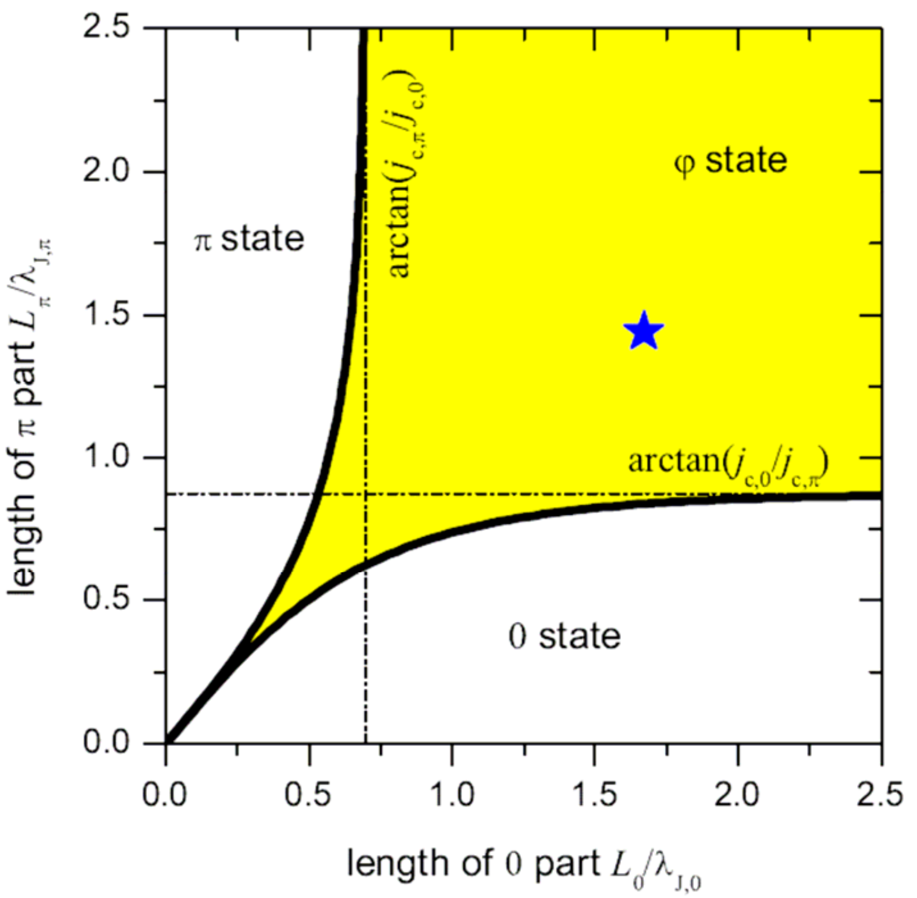


@ T = 2.35 K

H. Sickinger et al., PRL **109**, 107002 (2012)

CPR: Analytics vs. numerics



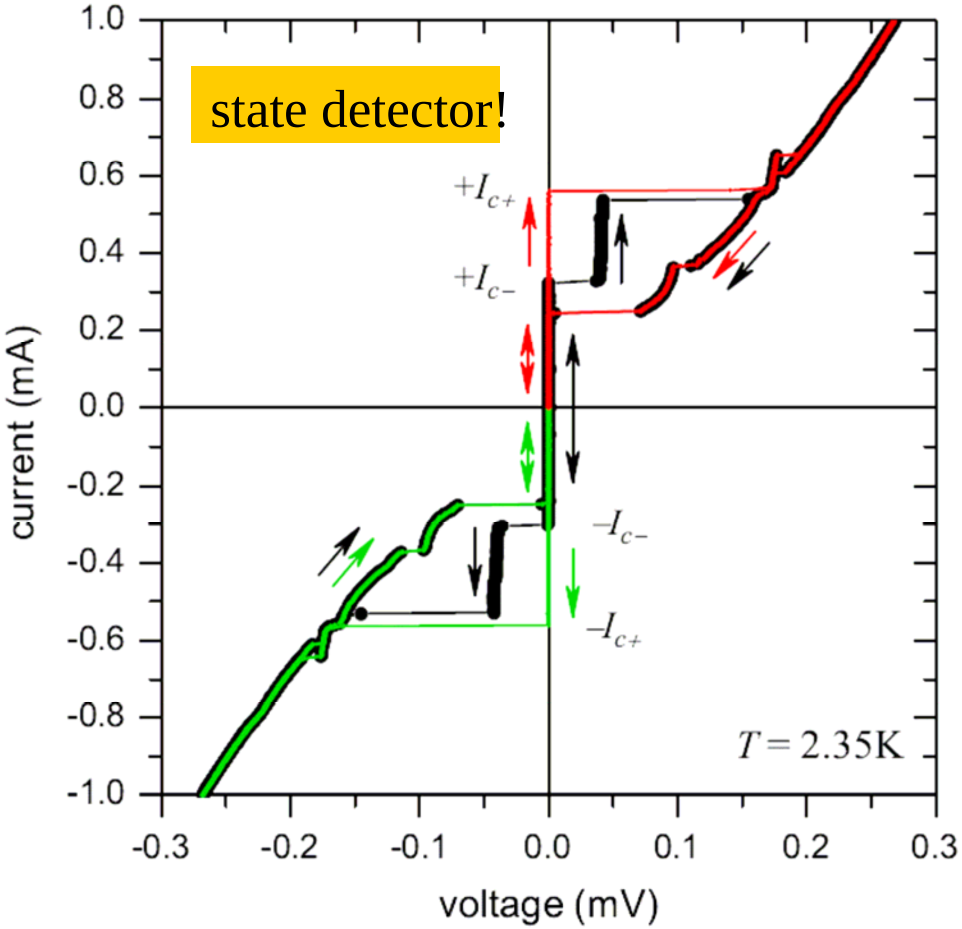


H. Sickinger et al., PRL **109**, 107002 (2012)

Experiment: *I-V* characteristic

Observation of I_{c+} and I_{c-}

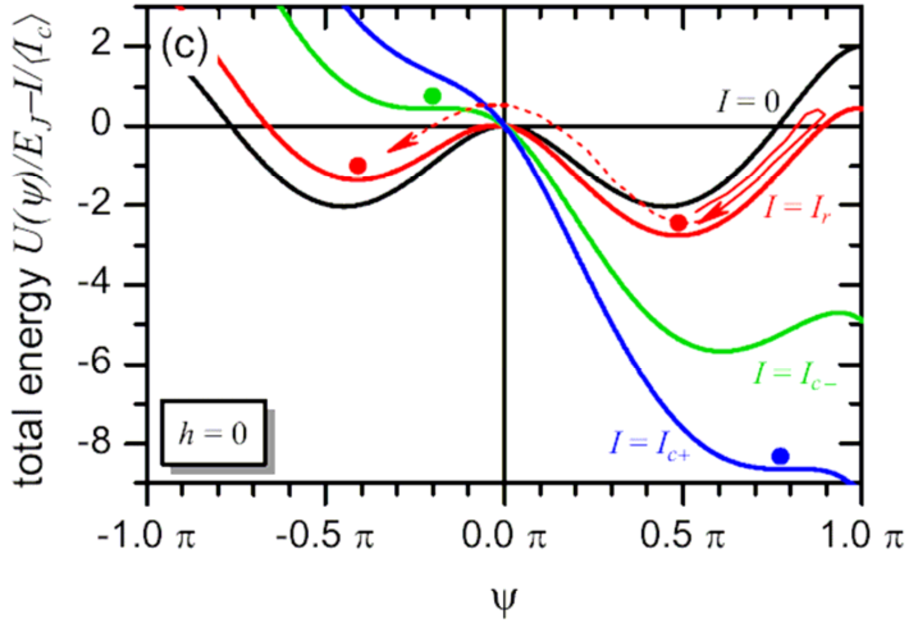
- I_{c+} is always observed
- I_{c-} only @ 0.3 K < T < 3.5 K (low α)



Preparation of the desired state −φ, +φ

.. by using a special sweep sequence [Goldobin et al., PRB **76**, 224523 (2007)]

- if I>0 and decreasing, we trap $\phi = +\varphi$ and should observe $+I_{c+}$ or $-I_{c-}$
- @ 300mK< T <2.3 K no determinism
- @ T > 2.3 K (high α) as predicted!



📖 H. Sickinger et al., PRL **109**, 107002 (2012)

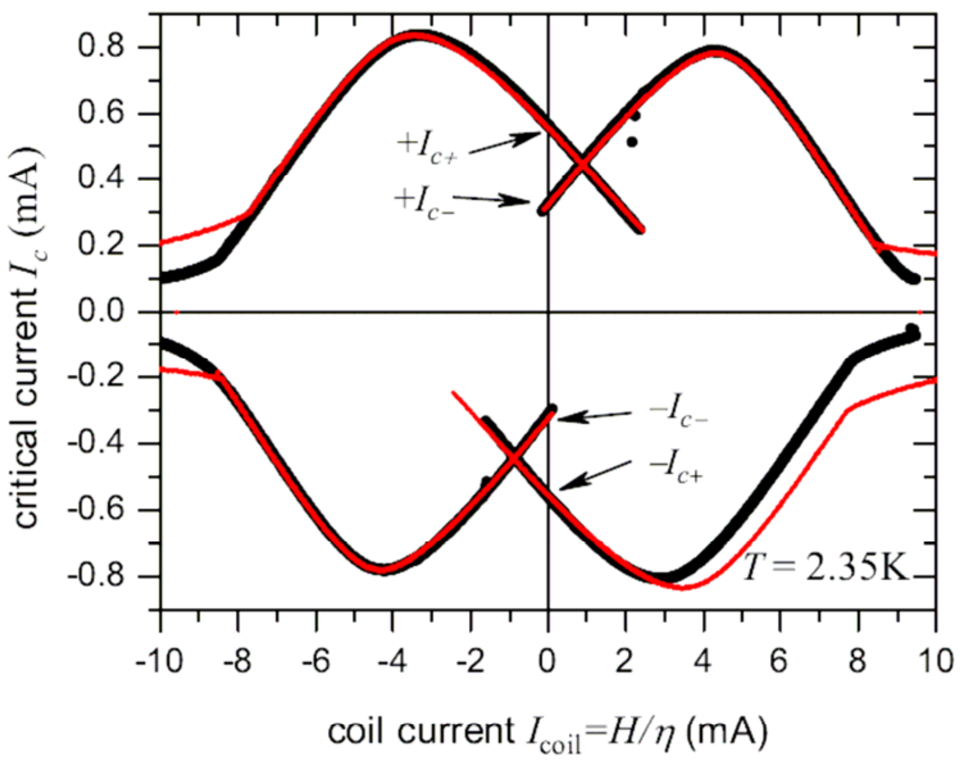
Experiment: $I_c(H)$

Observation of shifted main minimum

- @ 300mK < T < 4.2K
- @ T = 2.35 K, $\varphi = 0.45\pi$

Observation of 2 branches crossing

- @ T < 3.5 K
- H_{reset} improves visibility
- L-branch = escape from $+\varphi$
- R-branch = escape from $-\varphi$



📖 H. Sickinger et al., PRL **109**, 107002 (2012)

φ JJ with tunable CPR: Summary

- Innitial question about $I_c(H)$ is resolved (shifted minimum=x-point)
- New: φ-JJ with cos(ψ) term in CPR (and ground state) tunable by magnetic field!
- New: First experimental evidence of φ-JJ in SIFS 0-π heterostructure:
  - the value of $\varphi \sim 0.45\pi$ is found from x-point position
  - two critical currents at H=0 (phase escapes from $+\varphi$ and from $-\varphi$ wells)
  - detection of state by measuring I_c
  - preparation of state ($+\varphi$ or $-\varphi$) by using magnetic field
  - preparation of state ($+\varphi$ or $-\varphi$) by using bias sweep sequence
- If you understood the idea, then do the home work:
  - calculate the effective CPR of the usual short JJ in magnetic field (use the linear phase approximation $\phi(x)=hx+\psi$, $-L/2< x <+L/2$)
  - calculate the effective CPR of the short 0-π JJ in magnetic field (in the linear phase approximation $\phi(x)=hx+\psi+\theta(x)$, $-L/2< x <+L/2$)

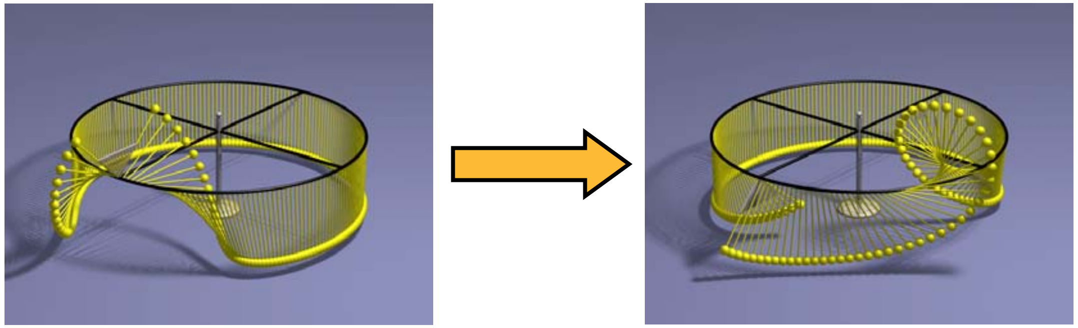
📖 E. Goldobin et al., PRL **107**, 227001 (2011)
📖 H. Sickinger et al., PRL **109**, 107002 (2012)

φ JJ -- home work

- If you understood the idea, then do the home work:
  - calculate the effective CPR of the usual short JJ in magnetic field (use the linear phase approximation $\phi(x)=hx+\psi$, $-L/2< x <+L/2$)
  - calculate the effective CPR of the short 0-π JJ in magnetic field (in the linear phase approximation $\phi(x)=hx+\psi+\theta(x)$, $-L/2< x <+L/2$)
  - calculate the loading capability of the φ JJ, which works as a phase battery. Assume that the JJ is in + φ state, connected to inductance *L* and calculate the current flowing (a) clockwise, (b) counterclockwise. When the battery dies out? What exactly happens in cases (a) and (b)?

📖 E. Goldobin et al., PRL **107**, 227001 (2011)
📖 H. Sickinger et al., PRL **109**, 107002 (2012)

Summary



- 0 JJ + π JJ = 0-π JJ.
- Technologies
  - SIFS 0-π JJs
  - s-wave/s-wave 0-π JJs
  - creating artificial phase discontinuities, 0-κ junction.
- Single fractional vortex
  - ground states
  - depinning by bias current, thermal escape, MQT
  - eigenmodes
- Fractional vortex molecules
  - ground states
  - rearrangement by bias current
  - eigenmodes splitting
- Composite φ Josephson (made of 0 and π pieces)
  - arbitrary phase battery
  - two-state system (in quantum domainm when states are coupled=TLS)
  - CPR tunable by magnetic field (e.g. manipulation)

Thanks for your attention

..and enjoy the conference!